Dear student following is a Moderate level [○○●○○] test paper. Score of 9 Marks in 10 Minutes would be a satisfactory performance. Questions 1-6 (+3, –1). (All questions have one option correct only.)

Q.1 If A and B are square matrices of same order and A is non-singular, then for a positive integer n $(A^{-1}BA)^n$ is equal to

(A) $A^{-n}B^nA^n$ (B) $A^nB^nA^{-n}$

(C) $A^{-1}B^nA$ (D) $n(A^{-1}BA)$

Q.2 If $3A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ x & 2 & y \end{bmatrix}$ and $A.\ A^t = I$,

then x + y equals to

(A) –1 (B) 1

(C) 2 (D) None of these

Q.3 The index of the matrix $\begin{pmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{pmatrix}$ is

(A) 2 (B) 3

(C) 4 (D) None of these

Q.4 $\begin{vmatrix} x^2+1 & 1 & x+1 \\ 2x^2-1 & 1 & x+2 \\ 3x^2-1 & 1 & x+3 \end{vmatrix} = a_0 + a_1 x + a_2 x^2 + a_3 x^3$ ...., then a_1 equal to

(A) 1 (B) 2

(C) 3 (D) 0

Q.5 If $f(x) = \begin{vmatrix} \cos(x+\alpha) & \cos(x+\beta) & \cos(x+\gamma) \\ \sin(x+\alpha) & \sin(x+\beta) & \sin(x+\gamma) \\ \sin(\beta-\gamma) & \sin(\gamma-\alpha) & \sin(\alpha-\beta) \end{vmatrix}$,

then $f(\theta) - 2f(\phi) + f(\psi)$ is equal to....

(A) 1 (B) –1

(C) 0 (D) 2

Q.6 If $f(x) = \begin{vmatrix} x+\alpha & x+c & x+a \\ x+b & x+\beta & x+a \\ x+b & x+b & x+\gamma \end{vmatrix}$, $g(x) = (\alpha-x)(\beta-x)(\gamma-x)$, then the value of $\frac{bg(a)-ag(b)}{b-a}$ is

(A) f (1) (B) (–1)

(C) f (0) (D) None of these

MATHEMATICS IIT JEE (06 / 06 / 2007) (MATRIX & DETERMINANT) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	3	○	○	○	○	5	○	○	○	○
2	○	○	○	○	4	○	○	○	○	6	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6
Ans.	C	D	B	D	C	C

SOLUTIONS

Sol.1 (C)

$(A^{-1} BA)^2 = (A^{-1} BA)(A^{-1} BA) = A^{-1} BA$

Similarly $(A^{-1} BA)^n = A^{-1} BA^n$.

Sol.2 (D)

$$AA^{-1} = \frac{1}{9}\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & -2\\ x & 2 & y\end{bmatrix}\begin{bmatrix}1 & 2 & x\\ 2 & 1 & 2\\ 2 & -2 & y\end{bmatrix} = \frac{1}{9}$$

$$\begin{bmatrix}9 & 0 & x+4+2y\\ 0 & 9 & 2x+2-2y\\ X+4+2y & 2X+2-2y & x^2+4+y^2\end{bmatrix} I$$

$$= \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$\Rightarrow \quad x = -2,\ y = -1$

$\Rightarrow \quad x + y = -3.$

Sol.3 (B)

$$A^2 = \begin{pmatrix}0 & 1 & 0\\ 3 & 3 & 9\\ -2 & -1 & -3\end{pmatrix}$$

$$A^3 = \begin{pmatrix}0 & 0 & 0\\ 3 & 3 & 9\\ -1 & -1 & -3\end{pmatrix}\begin{pmatrix}1 & 1 & 3\\ 5 & 2 & 6\\ -2 & -1 & -3\end{pmatrix}$$

$$\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$$

So index is 3.

Sol.4 (D)

$$a_1 = f'(0) \text{ where } f(x)\begin{vmatrix}x^2+1 & 1 & x+1\\ 2x^2-1 & 1 & x+2\\ 3x^2-1 & 1 & x+3\end{vmatrix}$$

$$f'(x) = \begin{vmatrix}2x & 1 & x+1\\ 4x & 1 & x+2\\ 6x & 1 & x+3\end{vmatrix} + 0 + 0 = 0$$

$(R_3 - R_2,\ R_2 - R_1)$

$\therefore\ f'(0) = 0 \Rightarrow a_1 = 0.$

Sol.5 (C)

Now,

$$f(x) = -\begin{vmatrix}\cos(x+\alpha) & \cos(x+\beta) & \cos(x+\gamma)\\ \sin(x+\alpha) & \sin(x+\beta) & \sin(x+\gamma)\\ \sin(\beta-\gamma) & \sin(\gamma-\alpha) & \sin(\alpha-\beta)\end{vmatrix}$$

$$+\begin{vmatrix}\cos(x+\alpha) & \cos(x+\beta) & \cos(x+\gamma)\\ \sin(x+\alpha) & \sin(x+\beta) & \sin(x+\gamma)\\ \sin(\beta-\gamma) & \sin(\gamma-\alpha) & \sin(\alpha-\beta)\end{vmatrix}$$

$= 0 + 0 = 0$

$\Rightarrow$ f(x) is a constant function, let f(x)= c

$\therefore f(\theta) - 2f(\phi) + f(\psi) = c - 2c + c = 0.$

Sol.6 (C)

$$\begin{vmatrix}x+\alpha & a-\alpha & 0\\ x+b & \beta-b & x+a\\ x+b & 0 & x+\gamma\end{vmatrix}\ \begin{pmatrix}C_3 \to C_3 - C_2\\ C_2 \to C_2 - C_1\end{pmatrix}$$

= a liner expression in x = px + q (say)

$f(-a) = -p\alpha - q : f(-b) = -pb + q$

$$f(-a) = \begin{vmatrix}\alpha-a & a-a & 0\\ b-a & \beta-b & a-\beta\\ b-a & 0 & \gamma-b\end{vmatrix}$$

$$= \begin{vmatrix}\alpha-a & a-\alpha & 0\\ b-a & \beta-b & a-\beta\\ b-a & 0 & \gamma-b\end{vmatrix}\ (C_2 \to C_2 + C_1) = (\alpha$$

$-a)\ (\beta-a)\ (\gamma-a) = g(a)$

$$f(-b) = \begin{vmatrix}\alpha-b & a-b & a-b\\ 0 & \beta-b & a-b\\ 0 & 0 & \gamma-b\end{vmatrix}$$

$= (\alpha-b)\ (\beta-b)\ (\gamma-b) = g(b)$

$\therefore\ g(a) = pa - q.g\ (b) = -pb + q$

$\because\ bg(a) - a\ g(b) = (b-a)\ f(0)$