Dear student following is a Moderate level [○ ○ ● ○ ○] test paper. Score of 18 Marks in 15 Minutes would be a satisfactory performance. Questions 1-10(+3,–1). (All questions have only one option correct).

Q.1 Let complex numbers $z_1 = 10 + 6i$ and $z_2 = 4 + 2i$. If $\arg\left(\frac{z - z_1}{z - z_2}\right) = \frac{\pi}{4}$, then the centre and radius of the locus of complex no. z are

(A) $5 - 7i, \sqrt{52}$ (B) $5 + 7i, \sqrt{26}$
(C) $5 - 7i, \sqrt{26}$ (D) $5 + 7i, \sqrt{52}$

Q.2 If z_1, z_2, z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = 1$, then $|z_1 - z_2|^2 + |z_2 - z_3|^2 + |z_3 - z_1|^2$ can not exceed

(A) 6 (B) 9 (C) 12 (D) None

Q.3 Dividing f(z) by z − i, we obtain the remainder i and dividing it by z + i, we get the remainder 1 + i. The remainder upon the division of f(z) by $z^2 + 1$ is

(A) $\frac{1}{2}(z + 1) + i$ (B) $\frac{1}{2}(iz + 1) + i$
(C) $\frac{1}{2}(iz - 1) + i$ (D) $\frac{1}{2}(z + i) + 1$

Q.4 For any two complex numbers z_1, z_2 and any real numbers a and b

$$|az_1 - bz_2|^2 + |bz_1 + az_2|^2 =$$

(A) $(a^2 - b^2)(|z_1|^2 + |z_2|^2)$
(B) $(a^2 + b^2)(|z_1|^2 + |z_2|^2)$
(C) $(b^2 - a^2)(|z_1|^2 + |z_2|^2)$
(D) None of these

Q.5 If the complex numbers z_1, z_2 and z_3 represent the vertices of an equilateral triangle such that $|z_1| = |z_2| = |z_3|$ then

(A) $z_1 + z_2 + z_3 \neq 0$ (B) $z_1 + z_2 + z_3 = 0$
(C) $z_1z_2 + z_2z_3 + z_3z_1 = 0$
(D) $z_1z_2 + z_2z_3 + z_3z_1 \neq 0$

Statement :

Let z be a complex number lying on a circle $|z| = \sqrt{2}\,a$ and $b = b_1 + ib_2$ (any complex number), then

Q.6 The equation of tangent at point 'b' is

(A) $z\bar{b} + \bar{z}b = a^2$ (B) $z\bar{b} + \bar{z}b = 2a^2$
(C) $z\bar{b} + \bar{z}b = 3a^2$ (D) $z\bar{b} + \bar{z}b = 4a^2$

Q.7 The length of perpendicular from z_0 on the tangent at 'b' is :

(A) $\frac{|z_0\bar{b} + \bar{z}_0 b - a^2|}{2\sqrt{2}a}$ (B) $\frac{|z_0\bar{b} + \bar{z}_0 b - 2a^2|}{2\sqrt{2}a}$
(C) $\frac{|z_0\bar{b} + \bar{z}_0 b - 3a^2|}{2\sqrt{2}a}$ (D) $\frac{|z_0\bar{b} + \bar{z}_0 b - 4a^2|}{2\sqrt{2}a}$

Q.8 The equation of straight line parallel to the tangent and passing through centre of circle is :

(A) $z\bar{b} + \bar{z}b = 0$ (B) $2z\bar{b} + \bar{z}b = \lambda$
(C) $2z\bar{b} + 3\bar{z}b = 0$ (D) $z\bar{b} + \bar{z}b = \lambda$

Q.9 The equation of lines passing through the centre of the circle and making an angle $\pi/4$ with the normal at 'b' are

(A) $z = \pm\frac{ib^2}{2a^2}\bar{z}$ (B) $z = \pm\frac{ib^2}{a^2}\bar{z}$
(C) $z = \pm\frac{ib^2}{3a^2}\bar{z}$ (D) $z = \pm\frac{ib^2}{4a^2}\bar{z}$

Q.10 If z_1, z_2 and z_3 are the vertices of an isosceles right angled triangle at the vertex z_2, then value $(z_1 - z_2)^2 + (z_2 - z_3)^2$ is

(A) −1 (B) 0
(C) $(z_1 - z_3)^2$ (D) None

MATHEMATICS IIT JEE (JULY 2nd WEEK CLASS TEST 3) (COMPLEX NUMBER) ANSWER KEY

Name : .... Roll No. : ....

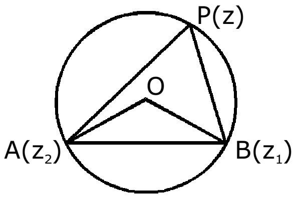
	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○
										10	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	B	B	B	B	B	D	D	A	A	B

SOLUTIONS

Sol.1 (B)



Let $O(z_0)$ be the centre of the circle

we have $\angle AOB = \dfrac{\pi}{2}$

$AB = |z_1 - z_2|$

$AB = |6 + 4i| = \sqrt{52}$

Let $OA = OB = r$

$\Rightarrow AB = r\sqrt{2}$

$\Rightarrow r = \sqrt{26}$

Also, $\dfrac{z_1 - z_0}{z_2 - z_0} = e^{i\pi/2} = i$

$\Rightarrow z_1 - z_0 = i(z_2 - z_0)$

$\Rightarrow z_0 = \dfrac{1}{2}(z_2 + iz_1 - iz_2 + z_1)$

$= 5 + 7i$

Sol.2 (B)

Let $y = |z_1 - z_2|^2 + |z_2 - z_3|^2 + |z_3 - z_1|^2$

$= (z_1 - z_2)(\bar{z}_1 - \bar{z}_2) + (z_2 - z_3)(\bar{z}_2 - \bar{z}_3) + (z_3 - z_1)(\bar{z}_3 - \bar{z}_1)$

$= 6 - (z_1\bar{z}_2 + z_2\bar{z}_1 + z_2\bar{z}_3 + z_3\bar{z}_2 + z_3\bar{z}_1 + z_1\bar{z}_3)$ ....(1)

Now we know $|z_1 + z_2 + z_3|^2 \geq 0$

$\Rightarrow 3 + (z_1\bar{z}_2 + z_2\bar{z}_1 + z_2\bar{z}_3 + z_3\bar{z}_2 + z_3\bar{z}_1 + z_1\bar{z}_3) \geq 0$ ...(2)

From (1) and (2)

$y \leq 9$.

Sol.3 (B)

$f(z) = g(z)(z - i)(z + i) + az + b$; $a, b \in C$

$f(i) = i \Rightarrow ai + b = i$ ..(1)

$f(-i) = 1 + i \Rightarrow a(-i) + b = 1 + i$ ..(2)

From (1) and (2)

$a = \dfrac{i}{2}$, $b = \dfrac{1}{2} + i$.

Hence required remainder =

$az + b = \dfrac{1}{2}iz + \dfrac{1}{2} + i$.

Sol.4 (B)

We know that

$|z_1 \pm z_2|^2 = |z_1|^2 + |z_2|^2 \pm 2Re(z_1\bar{z}_2)$

$\therefore |az_1 - bz_2|^2 = a^2|z_1|^2 + b^2|z_2|^2 - 2abRe(z_1\bar{z}_2)$

and $|bz_1 - az_2|^2 = b^2|z_1|^2 + a^2|z_2|^2 + 2abRe(z_1\bar{z}_2)$

Now adding two expressions, we get

$|az_1 - bz_2|^2 + |bz_1 + az_2|^2 =$

$(a^2 + b^2)(|z_1|^2 + |z_2|^2)$

Sol.5 (B)

$\because$ z_1, z_2 and z_3 are the vertices of an equilateral triangle and $|z_1| = |z_2| = |z_3|$. Origin is the circumcentre of the triangle since in an equilateral triangle the circumcentre and the centroid coincide, therefore, we must have

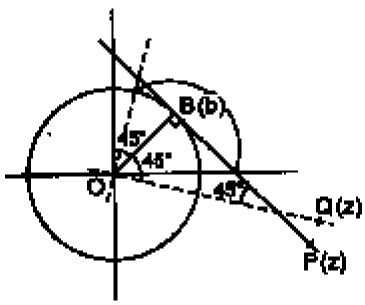
$\dfrac{1}{3}(z_1 + z_2 + z_3) = 0$

$\Rightarrow z_1 + z_2 + z_3 = 0$

Sol. 6 - 9

6 - (D), 7 - (D), 8 - (A), 9 - (A)

Let P(z) be any point on the tangent at B(b). Then



$OB \perp PB$.

$\Rightarrow \dfrac{z - b}{|z - b|} = \dfrac{b - 0}{|b - 0|}e^{i\pi/2}$

$\Rightarrow \dfrac{z - b}{|z - b|} = \dfrac{b}{|b|}i$

$\Rightarrow \dfrac{(z - b)^2}{(z - b)(\bar{z} - \bar{b})} = -\dfrac{b}{|b|^2}$

$\Rightarrow \dfrac{(z - b)}{\bar{z} - \bar{b}} = -\dfrac{b}{\bar{b}}$

$\Rightarrow \quad z\bar{b} - b\bar{b} = -\bar{z}b + b\bar{b}$

$\Rightarrow \quad z\bar{b} + \bar{z}b = 2b\bar{b}$

$\Rightarrow \quad z\bar{b} + \bar{z}b = 2|b|^2$

{$\because$ be lie on $z = \sqrt{2}$ a $\therefore$ $|b| = \sqrt{2}$ a}

$\Rightarrow \quad z\bar{b} + \bar{z}b = 4a^2$ .....(i)

also length of perpendicular from z_0 on (i) is,

$\Rightarrow \quad \dfrac{|z_0\bar{b} + \bar{z}_0 b - 4a^2|}{2|b|} \Rightarrow \dfrac{|z_0\bar{b} + \bar{z}_0\bar{b} - 4a^2|}{2\sqrt{2}a}$

{$\because$ b lie on $z = \sqrt{2}$ a $\therefore$ $|b| = \sqrt{2}$ a}

Also, equation of straight line parallel to (i) is, $z\bar{b} + \bar{z}b = \lambda$, which passes through origin.

$\therefore \quad \lambda = 0$

or $z\bar{b} + \bar{z}b = 0$ is a straight line parallel to tangent at 'b' and passing through centre again, let Q(z) on required lines, then

$$\frac{z}{|z|} = \frac{b}{|b|} e^{\pm i\pi/4}$$

$$\Rightarrow \quad \frac{z^2}{|z|^2} = \frac{b^2}{|b|^2} . e^{\pm i\pi/2}$$

$$\Rightarrow \quad \frac{z^2}{z.\bar{z}} = \frac{b^2}{b.\bar{b}} (\pm i)$$

$$\Rightarrow \quad z = \pm \left(\frac{b}{\bar{b}} \bar{z}\right) i$$

$$\Rightarrow \quad z = \pm i \frac{b^2}{|\bar{b}|^2} \bar{z}$$

$$\Rightarrow \quad z = \pm i \frac{b^2}{2a^2} \bar{z}$$

Hence, the required lines are

$$z = \pm i \frac{b^2}{2a^2} \bar{z}$$

Sol.10 (B)

As ABC is an isosceles right angled triangle with right angle at B.

$\Rightarrow$ BA = BC and $\angle$ABC = 90º

$\Rightarrow$ $|z_1 - z_2| = |z_3 - z_2|$

and $\arg\left(\dfrac{z_3 - z_2}{z_1 - z_2}\right) = \dfrac{\pi}{2}$ ...(i)

$$\Rightarrow \quad \frac{z_3 - z_2}{z_1 - z_2} = \left|\frac{z_3 - z_2}{z_1 - z_2}\right| (\cos \pi/2 + i \sin \pi/2)$$

$\Rightarrow \quad \dfrac{z_3 - z_2}{z_1 - z_2} = i$ (using (i))

$\Rightarrow$ $(z_3 - z_2)^2 = i^2 (z_1 - z_2)^2$

$\Rightarrow$ $(z_3 - z_2)^2 + (z_1 - z_2)^2 = 0$