Dear student following is a Moderate level [○ ○ ● ○ ○] test paper. Score of 18 Marks in 15 Minutes would be a satisfactory performance. Questions 1-10(+3,–1). (All questions have only one option correct).

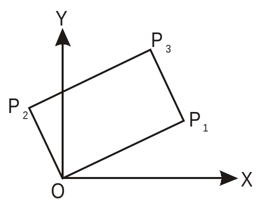
Q.1 The value of $i^{1+3+5+\ldots\ldots+(2n+1)}$ is-
(A) i if n is even, – i if n is odd
(B) 1 if n is even, – 1 if n is odd
(C) 1 if n is odd, – 1 if n is even
(D) i if n is even, – 1 if n is odd

Q.2 If z = 1 + i, then the multiplicative inverse of z^2 is (where $i = \sqrt{-1}$)
(A) 2i (B) 1 – i
(C) – i/2 (D) i/2

Q.3 If z_1, z_2 are two complex numbers such that $\left|\frac{z_1 - z_2}{z_1 + z_2}\right| = 1$ and $iz_1 = kz_2$, where $k \in R$, then the angle between $z_1 - z_2$ and $z_1 + z_2$ is-
(A) $\tan^{-1}\left(\frac{2k}{k^2+1}\right)$ (B) $\tan^{-1}\left(\frac{2k}{1-k^2}\right)$
(C) $-2\tan^{-1} k$ (D) $2\tan^{-1} k$

Q.4 The vector z = 3 – 4i is turned anticlockwise through an angle of 180º and stretched 2.5 times. The complex number corresponding to the newly obtained vector is-
(A) $\frac{15}{2} - 10i$ (B) $\frac{-15}{2} + 10i$
(C) $\frac{-15}{2} - 10i$ (D) None of these

Q.5 If the points P_1 and P_2 represent two complex numbers z_1 and z_2, then the point P_3 represents the number



(A) $z_1 + z_2$ (B) $z_1 - z_2$
(C) $z_1 \times z_2$ (D) $z_1 \div z_2$

Q.6 If the roots of $z^3 + iz^2 + 2i = 0$ represent the vertices of a ΔABC in the Argand plane, then the area of the triangle is-
(A) $\frac{3\sqrt{7}}{2}$ (B) $\frac{3\sqrt{7}}{4}$ (C) 2 (D) None

Q.7 Let z_1 and z_2 be non-zero complex numbers satisfying the equation $z_1^2 - 2z_1z_2 + 2z_2^2 = 0$. The geometrical nature of the triangle whose vertices are the origin and the points representing z_1 and z_2 is-
(A) An isosceles right angled triangle
(B) A right angled triangle
(C) An equilateral triangle
(D) None of these

Q.8 The locus of the centre of a circle which touches the circle $|z - z_1| = a$ and $|z - z_2| = b$ externally (z, z_1 & z_2 are complex numbers) will be-
(A) An ellipse (B) A hyperbola
(C) A circle (D) None of these

Q.9 Let z and w be complex numbers such that $\bar{z} + i\bar{w} = 0$ and arg zw = π. Then arg z equals-
(A) $\frac{5\pi}{4}$ (B) $\frac{\pi}{2}$ (C) $\frac{3\pi}{4}$ (D) $\frac{\pi}{4}$

Q.10 Let z_1 and z_2 be two roots of the equation $z^2 + az + b = 0$, z being complex. Further, assume that the origin, z_1 and z_2 form an equilateral triangle. Then-
(A) $a^2 = 4b$ (B) $a^2 = b$
(C) $a^2 = 2b$ (D) $a^2 = 3b$

MATHEMATICS IIT JEE (JULY 2nd WEEK CLASS TEST 4) (COMPLEX NUMBER) ANSWER KEY

Name : .... **Roll No. :** ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○
										10	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	C	C	C	B	A	C	A	B	C	D

SOLUTIONS

Sol.1 (C)

Let $z = i^{[1+3+5+\ldots\ldots+(2n+1)]}$

Clearly series is A.P. with common difference = 2

$\because$ $T_n = 2n - 1$ and $T_{n+1} = 2n + 1$

So, number of terms in A.P. = n +1

Now, $S_{n+1} = \frac{n+1}{2}[2.1 + (n + 1 - 1)2]$

$\Rightarrow S_{n+1} = \frac{n+1}{2}[2 + 2n] = (n + 1)^2$

i.e. $i^{(n+1)^2}$

Now put n = 1, 2, 3, 4, 5, ...

n = 1, $z = i^4 = 1$, n = 2, $z = i^6 = -1$,

n = 3, $z = i^8 = 1$, n = 4, $z = i^{10} = -1$,

n = 5, $z = i^{12} = 1$, ...

Sol.2 (C)

Given z = 1 + i and $i = \sqrt{-1}$. Squaring both sides, we get $z^2 = (1 + i)^2 = 1 + 2i + i^2 = 1 + 2i - 1$ or $z^2 = 2i$.

Since it is multiplicative identity, therefore multiplicative inverse of $z^2 = \frac{1}{2i} \times \frac{i}{i} = \frac{i}{2i^2}$

$= -\frac{i}{2}$.

Sol.3 (C)

$\left|\frac{z_1 - z_2}{z_1 + z_2}\right| = 1 \Rightarrow \frac{z_1 - z_2}{z_1 + z_2} = \cos\alpha + i\sin\alpha$

$\Rightarrow \frac{2z_1}{-2z_2} = \frac{\cos\alpha + i\sin\alpha + 1}{\cos\alpha - 1 + i\sin\alpha}$

[Applying componendo and dividendo]

$\Rightarrow \frac{z_1}{z_2} = i\cot\frac{\alpha}{2}$

$\Rightarrow iz_1 = -\left(\cot\frac{\alpha}{2}\right)z_2$

But $iz_1 = kz_2 \Rightarrow k = -\cot\frac{\alpha}{2}$

Now $k = -\cot\frac{\alpha}{2} \Rightarrow \cot\frac{\alpha}{2} = -k$

$\Rightarrow \tan\alpha = \frac{+2k}{k^2 - 1}$

$\Rightarrow \tan\alpha = \frac{-2k}{1 - k^2}$

$\Rightarrow \alpha = \tan^{-1}\left(\frac{-2k}{1-k^2}\right) = -2\tan^{-1}k$

Now $\frac{z_1 - z_2}{z_1 + z_2} = \cos\alpha + i\sin\alpha$

$\Rightarrow$ α is the angle between $z_1 - z_2$ and $z_1 + z_2$.

Sol.4 (B)

3 – 4i i.e., (3, –4) lie in fourth quadrant in complex plane, after turned anticlockwise through 180º this will lie in II quadrant, therefore, the number will be –3 + 4i, now after stretching it 2.5 times i.e., multiplying by 2.5, the required complex number will be

$\frac{-15}{2} + 10i$.

Sol.5 (A)

This is a parallelogram $OP_1P_2P_3$. Then the mid point of P_1P_2 and OP_3 are the same. But midpoint of P_1P_2 is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

So that the coordinates of P_3 are $(x_1 + x_2, y_1 + y_2)$

Thus the point P_3 corresponds to sum of the complex number z_1 and z_2.

$\vec{OP_3} = \vec{OP_1} + \vec{P_1P_3} = \vec{OP_1} + \vec{OP_2} = z_1 + z_2$

Sol.6 (C)

$z^3 + iz^2 + 2i = (z - i)(z^2 + 2iz - 2) = 0$

$\Rightarrow z = i, 1 - i, -1 - i$

Let A = (0, 1), B = (1, – 1), C = (–1, –1)

$\therefore \Delta ABC = \frac{1}{2}\begin{vmatrix} 0 & 1 & 1 \\ 1 & -1 & 1 \\ -1 & -1 & 1 \end{vmatrix} = \frac{1}{2}|-2-2| = 2$

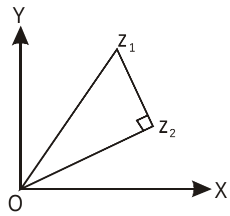
Sol.7 (A)

$z_1^2 - 2z_1z_2 + 2z_2^2 = 0$

$\Rightarrow \left(\frac{z_1}{z_2}\right)^2 - 2\left(\frac{z_1}{z_2}\right) + 2 = 0$

$\Rightarrow \frac{z_1}{z_2} = \frac{2 \pm \sqrt{4-8}}{2} = (1 \pm i)$

$\Rightarrow |z_1| = \sqrt{2}\,|z_2|$



Again, $z_1 = z_2 \pm iz_2 \Rightarrow z_1 - z_2 = \pm iz_2$

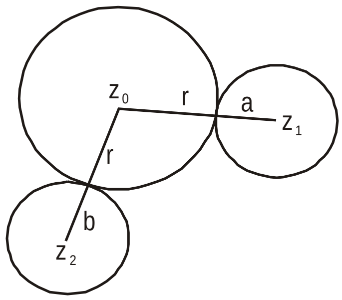
$\Rightarrow (z_1 - z_2)$ is $\perp$ to z_2 .... (2)

and $|z_1 - z_2| = |z_2|$

$\Rightarrow$ O, z_1 and z_2 form an isosceles right angled triangle.

Sol.8 (B)

Let the variable circle be $|z - z_0| = r$



Then $|z_0 - z_1| = a + r$ and $|z_0 - z_2| = b + r$

Eliminating r, we get,

$|z_0 - z_1| - |z_0 - z_2| = a - b$

Hence locus of z_0 is a hyperbola.

Sol.9 (C)

$\arg zw = \pi \Rightarrow \arg z + \arg w = \pi$ ....(1)

$\bar{z} + i\bar{w} = 0 \Rightarrow \bar{z} = -i\bar{w}$

$\therefore z = iw \Rightarrow \arg z = \frac{\pi}{2} + \arg w$

$\Rightarrow \arg z = \frac{\pi}{2} + \pi - \arg z$ [from (1)]

$\therefore \arg z = \frac{3\pi}{4}$

Sol.10 (D)

$z^2 + az + b = 0$; $z_1 + z_2 = -a$ & $z_1z_2 = b$

0, z_1, z_2 form an equilateral Δ

$\therefore$ $0^2 + z_1^2 + z_2^2 = 0.z_1 + z_1.z_2 + z_2.0$

(for an equilateral triangle,

$z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_2z_3 + z_3z_1$)

$\Rightarrow$ $z_1^2 + z_2^2 = z_1z_2$

$\Rightarrow$ $(z_1 + z_2)^2 = 3z_1z_2$

$\therefore$ $a^2 = 3b$