Dear student following is a Moderate level [○ ○ ● ○ ○] test paper. Score of 15 Marks in 10 Minutes would be a satisfactory performance. Questions 1-9(+3,–1) (All questions have only one option correct).

Passage :

Let $\lambda \in C$ and $z_1, z_2, z_3 \in C$ are such that

$$\frac{z_3 - z_1}{z_2 - z_1} = \lambda$$

Q.1 If λ is a given purely imaginary then all the triangles with vertices z_1, z_2, z_3 are

(A) similar (B) acute angled
(C) equilateral (D) obtuse angled

Q.2 Value of λ for which triangle with vertices z_1, z_2, z_3 is equilateral, is

(A) $\frac{1}{2}\left(1-\sqrt{3}i\right)$ (B) $\frac{1}{2}\left(1+\sqrt{3}i\right)$
(C) $\sqrt{3}i$ (D) $-\sqrt{3}i$

Q.3 If $\lambda \in \mathbf{R}$, then z_1, z_2, z_3

(A) lie on a circle
(B) are vertices of a right triangle
(C) lie on a straight line
(D) lie on a parabola

Q.4 If $\lambda = it$ ($t \in \mathbf{R}$) and z_2, z_3 are fixed, then locus of z_1 is

(A) a circle (B) an ellipse
(C) a straight line
(D) a pair of straight lines

Q.5 If $\lambda = e^{it}$ ($t \in \mathbf{R}$) and z_2, z_3 are fixed, then locus of z_1 is

(A) a circle (B) an ellipse
(C) a straight line
(D) a pair of straight lines

The following 3 questions consist of two statements one labelled Assertion (A) and the another labelled Reason (R). Select the correct answers to these questions using the codes given below :

(A) Both A and R are true and R is the correct explanation of A.
(B) Both A and R are true but R is not correct explanation of A
(C) A is true but R is false
(D) A is false but R is true.

Q.6 **Assertion** : If $\left|\frac{zz_1 - z_2}{zz_1 + z_2}\right| = k$, ($z_1, z_2 \neq 0$), then locus of z is circle.

Reason : As, $\left|\frac{z - z_1}{z - z_2}\right| = \lambda$, represents a circle if, $\lambda \notin \{0, 1\}$.

Q.7 **Assertion** : The equation $|z - i| + |z + i| = k$, $k > 0$, can represent an ellipse, if $k > 2i$

Reason : $|z - z_1| + |z - z_2| = k$, represents ellipse, if $|k| > |z_1 - z_2|$.

Q.8 **Assertion** : if $|z| = \max.\{|z - 1|, |z + 1|\}$, then $|z + \bar{z}| = 1$

Reason : $\max\{a, b\} = \begin{cases} a, & a > b \\ b, & b > a \end{cases}$

Q.9 An equation of straight line joining the complex numbers a and ib (where $a, b \in R$ and $a, b \neq 0$) is-

(A) $z\left(\frac{1}{a} - \frac{i}{b}\right) + \bar{z}\left(\frac{1}{a} + \frac{i}{b}\right) = 2$
(B) $z(a - ib) + \bar{z}(a + ib) = 2(a^2 + b^2)$
(C) $z(a + ib) + \bar{z}(a - ib) = 2ab$
(D) None of these

MATHEMATICS IIT JEE (JULY 3rd WEEK CLASS TEST 1) (COMPLEX NUMBER) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9
Ans.	A	B	C	A	C	D	D	B	A

SOLUTIONS

Sol.1 **(A)**

Let $\lambda = ia$ where $a \in \mathbf{R}$

Let z_1, z_2, z_3 and w_1, w_2, w_3 be vertices of two triangles ABC and PQR respectively, such that

$$\frac{z_3 - z_1}{z_2 - z_1} = \frac{w_3 - w_1}{w_2 - w_1} = ia \quad ....(1)$$

$$\Rightarrow \frac{|z_3 - z_1|}{|z_2 - z_1|} = \frac{|w_3 - w_1|}{|w_2 - w_1|} = |a|$$

$$\Rightarrow \frac{AC}{AB} = \frac{PR}{PQ} = |a| \quad ...(2)$$

Also, $\arg\left(\frac{z_3 - z_1}{z_2 - z_1}\right) = \arg\left(\frac{w_3 - w_1}{w_2 - w_1}\right) = \frac{\pi}{2}$

$$\Rightarrow \angle BAC = \angle QPR = \frac{\pi}{2}$$

From (2) and (3) we get ΔABC and ΔPQR are similar.

Sol.2 **(B)**

$$\lambda = \frac{z_3 - z_1}{z_2 - z_1} = \frac{|z_3 - z_1|}{|z_2 - z_1|}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$

$$= \frac{1}{2}\left(1 + \sqrt{3}i\right)$$

Sol.3 **(C)**

$z_3 - z_1 = \lambda(z_2 - z_1)$

$\Rightarrow$ $z_3 = (1 - \lambda)z_1 + \lambda z_2$

$\Rightarrow$ z_3 lies on the line joining z_1 and z_2.

Sol.4 **(A)**

As $\frac{z_3 - z_1}{z_2 - z_1}$ is purely imaginary angle between the segment joining z_3z_1 and z_2z_1 is a right angle.

$\Rightarrow$ z_1 lies on a circle with the segment joining z_2z_3 as diameter.

Sol.5 **(C)**

$$\left|\frac{z_3 - z_1}{z_2 - z_1}\right| = |e^{it}| = 1$$

$\Rightarrow$ z_1 is equidistant from z_2 and z_3.

$\Rightarrow$ z_1 lies on perpendicular bisector of the segement joining z_2 and z_3.

Sol.6 **(D)**

$$\left|\frac{zz_1 - z_2}{zz_1 + z_2}\right| = k \quad \Rightarrow \quad \left|\frac{z - \frac{z_2}{z_1}}{z + \frac{z_2}{z_1}}\right| = k$$

Clearly, if $k \neq 0, 1$ then z would lie on a circle. If k = 1, z would lie on a perpendicular bisector of line segment only $\frac{z_2}{z_1}$ and $\frac{-z_2}{z_1}$ represents a point. if k = 0.

Sol.7 **(D)**

As, we know $|z - z_1| + |z - z_2| = k$ represents an ellipse, if $|k| > |z_1 - z_2|$

Thus, $|z - i| + |z + i| = k$ represents ellipse, if

$|k| > |i + i|$

or $|k| > 2$

Sol.8 **(B)**

As, max. $\{|z - 1|, |z + 1|\}$

$$= \begin{cases} |z-1|, & \text{when } |z-1| > |z+1| \\ |z+1|, & \text{when } |z+1| > |z-1| \end{cases}$$

$\therefore$ if, $|z| = |z - 1|$

$\Rightarrow$ $|z|^2 = |z - 1|^2$

$\Rightarrow$ $z.\bar{z} = (z - 1)(\bar{z} - 1)$

$\Rightarrow$ $z + \bar{z} = 1$

Again if, $|z| = |z + 1|$

$\Rightarrow$ $|z|^2 = |z + 1|^2$

$\Rightarrow$ $z.\bar{z} = (z + 1)(\bar{z} + 1)$

$\Rightarrow$ $z + \bar{z} = -1$

Therefore $|z + \bar{z}| = 1$.

Q.9 An equation of straight line joining z_1 and z_2 is

$$\begin{vmatrix} z & \bar{z} & 1 \\ z_1 & \bar{z}_1 & 1 \\ z_2 & \bar{z}_2 & 1 \end{vmatrix} = 0$$

$\therefore$ equation of straight line joining a and ib is

$$\begin{vmatrix} z & \bar{z} & 1 \\ a & b & 1 \\ ib & -ib & 1 \end{vmatrix} = 0$$

$$\Rightarrow z(a + ib) - \bar{z}(a - ib) - 2iab = 0$$

$$\Rightarrow z\left(\frac{1}{a} - \frac{i}{b}\right) + \bar{z}\left(\frac{1}{a} + \frac{i}{b}\right) = 2$$