Dear student following is a Moderate level [○ ○ ● ○ ○] test paper. Score of 15 Marks in 10 Minutes would be a satisfactory performance. Questions 1-8(+3,–1) & 9(+6, 0) (All questions have only one option correct).

Q.1 Let A and B be complex numbers such that $\frac{A}{B} + \frac{B}{A} = 1$. Then the origin and two points A and B form a triangle which is-
(A) Equilateral
(B) Obtuse angled triangle
(C) Right angled triangle
(D) None of these

Q.2 If $\sin^{-1}\left(\frac{2(z-1)}{(1+i)^2}\right)$ is defined for some z, where z is non-real, then-
(A) Re(z) = 1, $-1 \le$ Im(z) ≤ 1
(B) Re(z) = 1, Im(z) = 2
(C) Re(z) = 1, $-\infty <$ Im(z) $< \infty$
(D) Re(z) = – 1, $|\text{Im}(z)| \le 1$

Q.3 The number of real roots of the equation $x^{12} - x^{11} + x^{10} - .... - x + 1 = 0$ is
(A) 2 (B) 6
(C) 12 (D) None of these

Q.4 If the roots of $(z-1)^{25} = 2\omega^2(z+1)^{25}$ (where ω is a complex cube root of unity) are plotted in the argand plane, they lie on-
(A) A straight line (B) A circle
(C) An ellipse (D) None of these

Q.5 The roots z_1, z_2, z_3 of the equation $x^3 + 3ax^3 + 2bx + c = 0$ ($a, b, c \in C$) form an equilateral triangle in the argand plane if and only if-
(A) $a^2 = b$ (B) $a = b^2$
(C) $a = \pm b$ (D) $|a| = |b|$

Q.6 If z_1, z_2, z_3 are three complex numbers such that $z_1^2 + z_2^2 + z_3^2 - z_2z_3 - z_3z_1 - z_1z_2 = 0$, then-
(A) z_1, z_2, z_3 must necessarily be equal
(B) At least two of z_1, z_2, z_3 must be equal
(C) One of z_1, z_2, z_3 must be zero
(D) None of these

Q.7 If $|z - i| = 1$ and arg $z = \theta$ where $\theta \in (0, \pi/2)$, then the value of $\cot\theta - \frac{2}{z}$ is-
(A) 2i (B) –i (C) i (D) 1 + i

Q.8 Let z_1 and z_2 be roots of the equation $z^2 + pz + q = 0$. where p and q may be complex numbers. Let A and B represents z_1 and z_2 in the complex plane, given $\angle AOB = \alpha \ne 0$ and OA = OB where O is the origin. Then what will be the value of p^2 ?
(A) $4q\cos\alpha/2$ (B) $4q\sin\alpha/2$
(C) $4q\cos^2\alpha/2$ (D) $4q\sin^2\alpha/2$

Q.9 Match the column
z lies on .... if

Column I	Column II
(i) $\|z-3\| + \|z-i\| = 10$	(a) circle
(ii) $\left\|\frac{2z-3}{z-i}\right\| = 2$	(b) hyperbola
(iii) $z^2 + \bar{z}^2 = 5$	(c) straight line
(iv) $\left\|\frac{z-6}{z-2i}\right\| = 3$	(d) ellipse

(A) (i) - (d), (ii) - (c), (iii) - (b), (iv) - (a)
(B) (i) - (a), (ii) - (d), (iii) - (b), (iv) - (c)
(C) (i) - (c), (ii) - (b), (iii) - (d), (iv) - (a)
(D) (i) - (d), (ii) - (a), (iii) - (c), (iv) - (b)

MATHEMATICS IIT JEE (JULY 3rd WEEK CLASS TEST 2) (COMPLEX NUMBER) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9
Ans.	B	A	D	B	A	D	C	C	A

SOLUTIONS

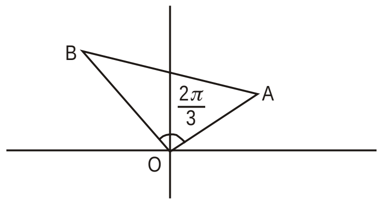
Sol.1 (B)

$\frac{A}{B} + \frac{B}{A} = 1 \Rightarrow z^2 - z + 1 = 0$, where $\frac{A}{B} = Z$

So, $z = \omega$ or ω^2

$\therefore \quad \frac{A}{B} = \omega$ or ω^2, so $\frac{B}{A} = \omega^2$ or ω

$\therefore \quad \left|\frac{A}{B}\right| = 1 \Rightarrow |A| = |B|$ arg $\left(\frac{A}{B}\right) = \frac{2\pi}{3}$



$\therefore \quad \Delta OAB$ is obtuse angled isoceles triangle

Sol.2 (A)

$\sin^{-1}\left(\frac{2(z-1)}{(1+i)^2}\right) = \sin^{-1}\left(\frac{2(z-1)}{1-1+2i}\right)$

$= \sin^{-1}\left(\frac{(z-1)}{i}\right) = \sin^{-1}\left(\frac{x+iy-1}{i}\right)$,

where $z = x + iy$

$= \sin^{-1}\left(\frac{x-1+iy}{i}\right) = \sin^{-1}\left(\frac{i(x-1+iy)}{-1}\right)$

$= \sin^{-1}\left(\frac{-y+i(x-1)}{-1}\right) = \sin^{-1}[-(x-1)i + y]$

$\Rightarrow \quad -(x-1)i + y$ must be a real number

$\Rightarrow \quad x - 1 = 0$ and $-1 \le y \le 1$

[as $\sin^{-1} x$ is defined if $-1 \le x \le 1$]

$\Rightarrow \quad x = 1$ and $-1 \le y \le 1$

$\Rightarrow \quad \text{Re}(z) = 1$ and $-1 \le \text{Im}(z) \le 1$

Sol.3 (D)

Multiplying the given equation by x + 1, the equation becomes $x^{13} + 1 = 0$

$\Rightarrow \quad x^{13} = -1$

$= \cos(2k+1)\pi + i\sin(2k+1)\pi$

$\Rightarrow \quad x = \cos\left(\frac{2k+1}{13}\right)\pi \pm i\sin\left(\frac{2k+1}{13}\right)\pi$

where k = 0, 1, 2, 3, 4, 5, 6

For k = 6, x = – 1 is not a root of given equation. Rest of the values are roots of given equation and none of them is real.

Sol.4 (B)

If z is a root of $(z-1)^{25} = 2\omega^2 (z+1)^{25}$, then

$\left(\frac{z-1}{z+1}\right)^{25} = 2\omega^2$

$\Rightarrow \quad \left|\frac{z-1}{z+1}\right|^{25} = 2|\omega^2| = 2$

$\Rightarrow \quad \left|\frac{z-1}{z+1}\right| = 2^{1/25}$

As $2^{1/25} \neq 1$, we get z lies on a circle.

Sol.5 (A)

z_1, z_2, z_3 form an equilateral triangle if and only if

$$\begin{vmatrix} 1 & z_2 & z_3 \\ 1 & z_3 & z_1 \\ 1 & z_1 & z_2 \end{vmatrix} = 0$$

$\Leftrightarrow \quad z_1^2 + z_2^2 + z_3^2 = z_2z_3 + z_3z_1 + z_1z_2$

$\Leftrightarrow \quad (z_1 + z_2 + z_3)^2 = 3(z_2z_3 + z_3z_1 + z_1z_2)$

$\Leftrightarrow \quad (-3a)^2 = 3(3b) \Leftrightarrow a^2 = b$

Sol.6 (D)

Multiplying the given expression by 2, we can write it as $(z_2 - z_3)^2+(z_3 - z_1)^2 + (z_1 - z_2)^2 = 0$

Taking $z_2 - z_3 = k$, $z_3 - z_1 = k\omega$, $z_1 - z_2 = k\omega^2$, (where $k \in C$) we can find infinite number of values of z_1, z_2 and z_3 satisfying the given relation.

Sol.7 (C)

We have $z - i = e^{i\alpha}$

$\Rightarrow \quad z = i + e^{i\alpha}$

$= i + \cos\alpha + i\sin\alpha$

$= \cos\alpha + i(1 + \sin\alpha)$

$\therefore \quad \theta = \tan^{-1}\left(\frac{1+\sin\alpha}{\cos\alpha}\right)$

$\Rightarrow \tan\theta = \frac{1+\sin\alpha}{\cos\alpha}$

$\cot\theta - \frac{2}{z} = \frac{\cos\alpha}{1+\sin\alpha}$

$- \frac{2}{\cos\alpha + i(1+\sin\alpha)}$

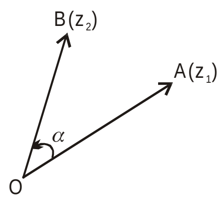
$= \frac{\cos\alpha}{1+\sin\alpha} - \frac{2[\cos\alpha - i(1+\sin\alpha)]}{\cos^2\alpha + (1+\sin\alpha)^2}$

$= \frac{\cos\alpha}{1+\sin\alpha} - \frac{2[\cos\alpha - i(1+\sin\alpha)]}{2(1+\sin\alpha)}$

$= \frac{i(1+\sin\alpha)}{1+\sin\alpha} = i$

Sol.8 (C)

Clearly $\vec{OB}$ is obtained by rotating $\vec{OA}$ through α



$\therefore \vec{OB} = \vec{OA}\, e^{i\alpha}$

$\Rightarrow z_2 - 0 = (z_1 - \theta)\, e^{i\alpha}$

$\Rightarrow \frac{z_2}{z_1} = \cos\alpha + i\sin\alpha$

$= 2\cos^2\frac{\alpha}{2} - 1 + 2i\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$

$\Rightarrow \frac{z_1+z_2}{z_1} = 2\cos\frac{\alpha}{2}\left(\cos\frac{\alpha}{2} + i\sin\frac{\alpha}{2}\right)$

$\Rightarrow \frac{(z_1+z_2)^2}{z_1^2} = 4\cos^2\frac{\alpha}{2}\left(\cos\frac{\alpha}{2} + i\sin\frac{\alpha}{2}\right)^2$

$= 4\cos^2\frac{\alpha}{2}(\cos\alpha + i\sin\alpha)$

$= 4\cos^2\alpha\left(\frac{z_2}{z_1}\right)$ {from (1)}

$\Rightarrow (z_1 + z_2)^2 = 4\cos^2\frac{\alpha}{2}(z_1z_2)$

$\Rightarrow (-p)^2 = 4\cos^2\frac{\alpha}{2}(q)$

($\because$ z_1 and z_2 are the roots of $z^2 + pz + q = 0$,
$\therefore z_1 + z_2 = -p,\ z_1z_2 = q$)

Therefore, $p^2 = 4q\cos^2\frac{\alpha}{2}$

Sol.9 (A)

(ii) can be written as

$\left|\frac{z - 3/2}{z - i}\right| = 1$

$\Rightarrow$ z lies on the perpendicular bisector of the segment joining 3/2 and i.

Now, Let $z = x + iy$

$\Rightarrow z^2 = x^2 - y^2 + 2ixy$

Thus, $z^2 + \bar{z}^2$

$\Rightarrow 2(x^2 - y^2) = 5$

$\Rightarrow x^2 - y^2 = \frac{5}{2}$

Thus, z lies on a hyperbola.