Dear student following is a Moderate level [○ ● ○] test paper. Score of 18 Marks in 15 Minutes would be a satisfactory performance. Questions 1-10 (+3, –1) (All questions have only one option correct)

Q.1 If n is a positive integer greater than 1, then $a - {}^nC_1(a-1) + {}^nC_2(a-2) - \ldots\ldots + (-1)^n(a-n)$ is equal to-

(A) n (B) a

(C) 0 (D) None of these

Q.2 The value of the sum of the series ${}^{14}C_0 \cdot {}^{15}C_1 + {}^{14}C_1 \cdot {}^{15}C_2 + {}^{14}C_2 \cdot {}^{15}C_3 + \ldots\ldots + {}^{14}c_{14} \cdot {}^{15}C_{15}$ is-

(A) ${}^{29}C_{12}$ (B) ${}^{29}C_{10}$ (C) ${}^{29}C_{14}$ (D) ${}^{29}C_{16}$

Q.3 If $(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots\ldots C_n x^n$, then $(C_0 + C_1)(C_1 + C_2) \ldots\ldots (C_{n-1} + C_n) = k \cdot C_1C_2C_3 \ldots\ldots C_n$ where k =

(A) $\frac{(n+1)^n}{n!}$ (B) $\frac{n^n}{(n-1)!}$

(C) $\frac{(n+1)^n}{(n-1)!}$ (D) None of these

Q.4 The coefficient of x^n in the expansion of $\frac{1+x-2x^3}{(1-x)^3}$ is-

(A) n (B) 3n – 1

(C) 4n – 1 (D) 5n – 1

Q.5 The greatest coefficient in the expansion of $(x + y + z + w)^{15}$ is-

(A) $\frac{15!}{3!\,(4!)^3}$ (B) $\frac{15!}{(3!)^3\,4!}$

(C) $\frac{15!}{2!\,(4!)^2}$ (D) None of these

Q.6 The expression $[x + (x^3 - 1)^{1/2}]^5 + [x - (x^3 - 1)^{1/2}]^5$ is a polynomial of degree-

(A) 5 (B) 6 (C) 7 (D) 8

Q.7 If the coefficient of x^2 in the expansion of $\frac{1}{(1+x)^2} + \frac{1}{(a+x)^2}$ is 246, then a =

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) None

Q.8 The number of irrational terms in the expansion of $(4^{1/5} + 7^{1/10})^{45}$ is-

(A) 40 (B) 5 (C) 41 (D) None

Q.9 The greatest integer which divides the number $101^{100} - 1$ is-

(A) 100 (B) 1000

(C) 10000 (D) 100000

Q.10 In the binomial expansion of $(a - b)^n$, $n \geq 5$, the sum of the 5th and 6th terms is zero. Then $\frac{a}{b}$ is equal to-

(A) $\frac{1}{6}(n-5)$ (B) $\frac{1}{5}(n-4)$

(C) $\frac{n}{n-4}$ (D) $\frac{6}{n-5}$

MATHEMATICS IIT JEE (OCT. 1st WEEK CLASS TEST 2) (BINOMIAL THEOREM) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○
										10	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	C	C	A	D	A	C	B	C	C	B

SOLUTIONS

Sol.1 (C)

We have, $t_{r+1} = (-1)^r \cdot {}^nC_r \cdot (a - r)$

$= (-1)^r \{a \cdot {}^nC_r - r\, {}^nC_r\}$

$= (-1)^r [a \cdot {}^nC_r - n \cdot {}^{n-1}C_{r-1}]$

$\{\because\ r \cdot {}^nC_r = n \cdot {}^{n-1}C_{r-1}\}$

Putting r = 0, 1, 2, ... , n we get

$t_1 = a \cdot {}^nC_0 - n \cdot 0$

$t_2 = -(a \cdot {}^nC_1 - n \cdot {}^{n-1}C_0)$

$t_3 = a \cdot {}^nC_2 - n \cdot {}^{n-1}C_1$

$t_4 = -(a \cdot {}^nC_3 - n \cdot {}^{n-1}C_2)$

....

$t_{n+1} = (-1)^n (a \cdot {}^nC_n - n \cdot {}^{n-1}C_{n-1})$

Adding, sum $= a[{}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + ... + (-1)^n \cdot {}^nC_n] + n[{}^{n-1}C_0 - {}^{n-1}C_1 + {}^{n-1}C_2 - ...... + (-1)^{n-1} \cdot {}^{n-1}C_{n-1}]$

$= a \times 0 + n \times 0 = 0$

Sol.2 (C)

We have,

$(1 + x)^{14} = {}^{14}C_0 + {}^{14}C_1x + {}^{14}C_2x^2 + ... + {}^{14}C_{14}x^{14}$ .... (1)

and $(x + 1)^{15} = {}^{15}C_0\, x^{15} + {}^{15}C_1x^{14} + {}^{15}C_2\, x^{13} + {}^{15}C_3\, x^{12} + ... + {}^{15}C_{15}$ .... (2)

Multiplying (1) and (2) and equating the co-efficient of x^{14}, we get

${}^{14}C_0 . {}^{15}C_1 + {}^{14}C_1 . {}^{15}C_2 + {}^{14}C_2 \cdot {}^{15}C_3 + ... + {}^{14}C_{14} \cdot {}^{15}C_{15}$

= the coefficient of x^{14} in

$(1 + x)^{29} = {}^{29}C_{14}$.

Sol.3 (A)

We have,

$C_0 + C_1 = {}^{n+1}C_1,\ C_1 + C_2 = {}^{n+1}C_2,\ ....\ C_{n-1} + C_n = {}^{n+1}C_n$

$\therefore$ The given expression $= {}^{n+1}C_1\ {}^{n+1}C_2 \cdot .... {}^{n+1}C_n$ .... (1)

Now, $${}^{n+1}C_r = \frac{(n+1)!}{r!\,(n-r+1)!}$$

$$= \frac{(n+1)\,n!}{r!\,(n-r+1)\,(n-r)!}$$

or $${}^{n+1}C_r = \frac{n+1}{n-r+1}\ {}^nC_r \qquad .... (2)$$

Putting n = 1, 2, 3, ... , n in (2), we get

$(C_0 + C_1)(C_1 + C_2) .... (C_{n-1} + C_n)$

$$= \frac{n+1}{n} C_1 \cdot \frac{n+1}{n-1} C_2 \ ..... \ \frac{n+1}{1} C_n$$

$$= \frac{(n+1)^n}{n!}\ C_1 C_2 C_3 \ .... \ C_n.$$

Sol.4 (D)

$$\frac{1+x-2x^3}{(1-x)^3} = \frac{1-x+2x-2x^3}{(1-x)^3}$$

$$= \frac{(1-x)+2x(1-x^2)}{(1-x)^3} = \frac{1+2x(1+x)}{(1-x)^2}$$

$= (1 + 2x + 2x^2)(1 - x)^{-2}$

$= (1 + 2x + 2x^2)$

$\{1 + 2x + 3x^2 + ... + (n + 1)\, x^n + ...\}$

$\therefore$ Coefficient of x^n = 1 × {coefficient of x^n in the bracket}

+ 2 × (coefficient of x^{n-1} in the bracket)

+ 2 (coefficient of x^{n-2} in the bracket)

$= 1 \cdot (n + 1) + 2 \cdot n + 2 \cdot (n - 1)$

$= n + 1 + 2n + 2n - 2 = 5n - 1.$

Sol.5 (A)

The greatest coefficient is

$$= \frac{n!}{(q!)^{k-r}\ [(q+1)!]^r}$$

[Here n = 15, q = 3, r = 3, k = 4]

Sol.6 (C)

$[x + (x^3 - 1)^{1/2}]^5 + [x - (x^3 - 1)^{1/2}]^5$

$= 2[{}^5C_0\, x^5 + {}^5C_2\, x^3 (x^3 - 1) + {}^5C_4\, x (x^3 - 1)^2]$

$= 2[x^5 + 10x^3 (x^3 - 1) + 5x (x^3 - 1)^2]$

$= 5x^7 + 10x^6 + x^5 - 10x^4 - 10x^3 + 5x$

which is polynomial of degree 7.

Sol.7 (B)

$$\frac{1}{(1+x)^2} + \frac{1}{(a+x)^2}$$

$= (1 + x)^{-2} + \frac{1}{a^2}\left(1+\frac{x}{a}\right)^{-2}$

$= (1 - 2x + 3x^2 - 4x^3 + ...)$

$+ \frac{1}{a^2}\left[1-\frac{2x}{a}+\frac{3x^2}{a^2}-\frac{4x^3}{a^3}+......\right]$

$\therefore$ Coefficient of $x^2 = 246$

$\Rightarrow 3 + \frac{1}{a^2}\left(\frac{3}{a^2}\right) = 246$

$\Rightarrow a^4 = \frac{1}{81}$; $a = \frac{1}{3}$ where $|x| < |a|$.

Sol.8 (C)

Total number of terms in the expansion of $(4^{1/5} + 7^{1/10})^{45}$ is 45 + 1 i.e. 46

The general term in the expansion is

$T_{r+1} = {}^{45}C_r \cdot 4^{\frac{45-r}{5}} \cdot 7^{\frac{r}{10}}$

T_{r+1} is rational if r = 0, 10, 20, 30, 40

$\therefore$ Number of rational terms = 5.

$\therefore$ Number of irrational terms = 46 – 5 = 41.

Sol.9 (C)

By Binomial theorem

$(1 + x)^n = \left[1+nx+\frac{n(n-1)}{2}.x^2.....x^n\right]$

or $(1 + x)^n - 1 = nx + \frac{n(n-1)}{2}x^2 ... x^n$

If x = n, $(1+n)^n - 1 = n^2 + \frac{n(n-1)}{2}n^2 ... n^n$

$(1 + n)^n - 1 = n^2\left[1+\frac{n(n-1)}{2}.....n^{n-2}\right]$

Put n = 100,

$(1 + 100)^{100} - 1$

$= (100)^2\left[1+\frac{100(100-1)}{2}....100^{98}\right]$

$(101)^{100} - 1$

$= (100)^2\left[1+\frac{100\times 99}{2}.......100^{98}\right]$

Clearly $(101)^{100} - 1$ is divisible by $(100)^2$ = 10000.

Sol.10 (B)

We have, $T_{r+1} = {}^nC_r\, a^{n-r} \cdot (-b)^r$

So, $t_5 + t_6 = 0$, gives

${}^nC_4\, a^{n-4} \cdot (-b)^4 + {}^nC_5\, a^{n-5}.(-b)^5 = 0$

$\Rightarrow \frac{n(n-1)(n-2)(n-3)}{24} \cdot a^{n-4} \cdot b^4$

$- \frac{n(n-1)(n-2)(n-3)(n-4)}{120} a^{n-5} b^5 = 0$

$\Rightarrow \frac{n(n-1)(n-2)(n-3)}{120}.a^{n-5} \cdot b^4$

$[5a - (n - 4)b] = 0$

$\Rightarrow 5a = (n - 4)b \quad \therefore \quad \frac{a}{b} = \frac{n-4}{5}$.