Q.1 The period of the function

$$f(x) = \frac{|\sin x| + |\cos x|}{|\sin x - \cos x|} \text{ is}$$

(A) $\pi/4$ (B) $\pi/2$
(C) π (D) 2π

Q.2 Which of the following function is surjective but not injective

(A) f: R $\to$ R $f(x) = x^4 + 2x^3 + 1$
(B) f: R $\to$ R $f(x) = x^3 + x + 1$
(C) f: R $\to$ R^+ $f(x) = \sqrt{1+x^2}$
(D) f: R $\to$ R $f(x) = x^3 + 2x^2 - x + 1$

Q.3 Suppose that f is continuous on [a,b] such that f (x) is an integer for each x in [a,b]. Then in [a,b]

(A) f is injective
(B) Range of f may have many elements
(C) {X} is zero for all $x \in [a, b]$ where denotes fractional part function
(D) f (x) is constant

Q.4 The domain of definition of the function f(x)=

$$\log_{\left[x+\frac{1}{x}\right]}\left|x^2 - x - 6\right| + {}^{16-x}C_{2x-1} + {}^{20-3x}P_{2x-5} \text{ is}$$

(Where [x] denotes greatest integer function.)

(A) {4, 5} (B) $\left[\frac{3}{4}, \infty\right] - \{2, 3\}$
(C) {2, 3} (D) $\left(-\frac{1}{4}, \infty\right)$

Q.5 Which of the following function (s) is/ are Transcidental ?

(C) $f(x) = 5\sin\sqrt{x}$

(B) $f(x) = \frac{2\sin 3x}{x^2 + 2x - 1}$

(C) $f(x) = \sqrt{x^2 + 2x + 1}$

(D) $f(x) = (x^2 + 3)\, 2^x$

Q.6 Which of following pairs of functions are identical :

(A) $f(x) = e^{\ell n \sec^{-1} x}$ & $g(x) = \sec^{-1}x$
(B) $f(x) = \tan(\tan^{-1} x)$&$g(x) = \cot(\cot^{-1})$
(C) f(x) = sgn (x) & g(x) = sgn (sgn (x))
(D) $f(x) = \cot^2 x.\ \cos^2 x$ & $g(x) = \cot^2 x - \cos^2 x$

Q.7 Which pairs (s) of function (s) is/are equal?

(A) $f(x) = \cos(2\tan^{-1}x)$; $g(x) = \frac{1-x^2}{1+x^2}$

(B) $f(x) = \frac{2x}{1+x^2}$; $g(x) = \sin(2\cot^{-1}x)$

(C) $f(x) = e^{\ell n(\operatorname{sgn} \cot^{-1} x)}$; $g(x) = e^{\ell n[1+\{x\}]}$

(D) $f(x) = \sqrt[x]{a}$, a>0; $g(x) = a^{\frac{1}{x}}$, a > 0

Where {x} and [x] denotes the fractional part & integral part functions.

MATHEMATICS IIT JEE (05 / 06 / 2007) (FUNCTIONS) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	⬭	⬭	⬭	⬭	4	⬭	⬭	⬭	⬭	7	⬭	⬭	⬭	⬭
2	⬭	⬭	⬭	⬭	5	⬭	⬭	⬭	⬭					
3	⬭	⬭	⬭	⬭	6	⬭	⬭	⬭	⬭					

ANSWER KEY

Que.	1	2	3	4	5	6	7
Ans.	C	D	D	A	A,B,D	B,C,D	A,B,C

SOLUTIONS

Sol.1 (C)

$$f\left(\frac{\pi}{2}+x\right) = \frac{|\cos x|+|\sin x|}{|\cos x+\sin x|} \neq f(x)$$

$f(\pi + x) = f(x)$

$f(2\pi + x) = f(x)$

So, period is 'π'.

Sol.2 (D)

(A) f(x) is into(Not surjective) and many one (Not injective)

(B) f(x) is onto (surjective) and one-one (injective)

(C) f(x) is into(Not surjective) and many one (Not injective)

(D) f(x) is onto (surjective) and many-one (not injective)

Sol.3 (D)

As f(x) is continuous and an integer so, f(x) is always an integer

$\Rightarrow$ f(x) = constant

Sol.4 (A)

$x^2 - x - 6 \neq 0$

$\Rightarrow (x - 3)(x + 2) \neq 0 \Rightarrow x \neq 3, -2$

and $\left[x+\frac{1}{x}\right] > 0$

$\Rightarrow x > 0$

and $16 - x \geq 2x - 1 \geq 0$

$\Rightarrow -\frac{1}{2} \leq x \leq \frac{17}{3}$

and (16 − x) and (2x − 1) are integers

So, x is of type (n) or $\left(n+\frac{1}{2}\right)$ when $n \in I$

Also $20 - 3x \geq 2x - 5 \geq 0 \Rightarrow \frac{5}{2} \leq x \leq 5$

and x must be integer.

$\Rightarrow$ Domain = {4, 5}

Sol.5 (A B D)

$f(x) = \sqrt{x^2+2x+1} = |x+1|$ is a polynomial function of order 1

So, not transcidental

Sol.6 (B C D)

For $f(x) = e^{\ell n(\sec^{-1} x)}$ defined $\Rightarrow \sec^{-1} x > 0$

$\Rightarrow \sec^{-1}x \neq 0 \quad \Rightarrow x \neq \pm 1$

but for $f(x) = \sec^{-1}x$, $|x| \geq 1$

Sol.7 (A B C)

For $f(x) = \sqrt[x]{a}$, a > 0 to be defined

iff x is only natural numbers $\Rightarrow$ Domain = N

but For $f(x) = a^{\frac{1}{x}}$, a > 0 defined for $x \in R$.