Dear student following is a Moderate level [O O ● O O] test paper. Score of 12 Marks in 10 Minutes would be a satisfactory performance. Questions 1-8 (+3, –1). (More than one options may be correct)

Q.1 The function defined by

$$f(x) = \begin{cases} x|x| & , \quad x \le -1 \\ [1+x]+[1-x] & , \quad -1 < x < 1 \\ -x|x| & , \quad x \ge 1 \end{cases} \text{ is-}$$

(A) An odd function
(B) An even function
(C) Neither even nor odd
(D) Even as well as odd

Q.2 Let f : [– 4, 4] ~ {– π, 0, π} → R, such that $f(x) = \cot(\sin x) + \left[\frac{x^2}{|a|}\right]$, where [.] denotes the greatest integer function, is an odd function. Complete set of values of 'a' is-
(A) (– 16, 16) – {0}
(B) (– ∞, – 16) ∪ (16, ∞)
(C) [– 16, 16] ~ {0}
(D) (– ∞, – 16) ∪ [16, ∞)

Q.3 Let f(x) = [x] and g(x) = |x|, then-
(A) (f + 2g) (– 1) = 1
(B) (f + 2g) (1) = 1
(C) $(gof - fog)\left(\frac{5}{3}\right) = 0$
(D) $(gof) - (fog)\left(-\frac{1}{5}\right) = 1$

Q.4 Let $f(x) = \sin^2\frac{x}{2} + \cos^2\frac{x}{2}$ and $g(x) = \sec^2 x - \tan^2 x$. Then two functions are equal over the set-
(A) φ (B) R
(C) $R - \left[x : x = (2n+1)\frac{\pi}{2}, n \in Z\right]$
(D) None of these

Q.5 If $f(x) = \cos^2 x + \cos^2\left(\frac{\pi}{3}+x\right) - \cos x . \cos\left(\frac{\pi}{3}+x\right)$, then which statement is incorrect-
(A) f(x) is an even function
(B) $f\left(\frac{\pi}{8}\right) = f\left(\frac{\pi}{9}\right)$
(C) f(x) is a constant function
(D) f(x) is a periodic function

Q.6 The graph of $f(x) = \left|\left|\frac{1}{|x|} - n\right| - n\right|$ lies in the (where n > 0)
(A) Ist and IIIrd quadrant
(B) IInd and IIIrd quadrant
(C) IIIrd and IVth quadrant
(D) Ist and IInd quadrant

Q.7 Let $f(x) = \cos\sqrt{p}\,x$, where p = [a] = the greatest integer less than or equal to a. If the period of f(x) is π, then-
(A) a ∈ [4, 5] (B) a = [4, 5]
(C) a ∈ [4, 5) (D) None of these

Q.8 If $f(x) = \left(\frac{x}{1-|x|}\right)^{1/2000}$, then D_f is-
(A) R – {– 1, 1} (B) (– ∞, 1)
(C) (– ∞, – 1] ∪ [0, 1) (D) Noen of these

MATHEMATICS IIT JEE (06 / 06 / 2007) (SETS, RELATIONS & FUNCTIONS) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○					

ANSWER KEY

Que.	1	2	3	4	5	6	7	8
Ans.	B	B	A,C,D	C	D	D	C	C

SOLUTIONS

Sol.1 (B)

$f(-x) = -x\,|-x|,\ -x \le -1$

$= -x\,|x|,\ x \ge 1$

$f(-x) = [1 - x] + [1 + x],\ -1 < -x < 1$

i.e., $1 > x > -1$

i.e., $-1 < x < 1$

$f(-x) = -(-x)\,|x|,\ -x \ge 1$

$= x\,|x|$ if $x \le -1$.

Thus f(x) is even function.

Sol.2 (B)

For f(x) to be odd, $\left[\dfrac{x^2}{|a|}\right]$ should not depend upon the value of x.

Since $x \in [-4, 4]$

$\therefore\ 0 \le x^2 \le 16$

$\Rightarrow\ \left[\dfrac{x^2}{|a|}\right] = 0$ if $|a| > 16$

$\Rightarrow\ a \in (-\infty, -16) \cup (16, \infty)$

Sol.3 (A, C, D)

$(f + 2g)(-1) = f(-1) + 2g(-1)$

$= [-1] + 2\,|-1| = -1 + 2 = 1$

$(f + 2g)(1) = f(1) + 2g(1)$

$= (1) + 2(1) = 1 + 2 = 3$

$(gof - fog)\left(\dfrac{5}{3}\right) = g\left(f\left(\dfrac{5}{3}\right)\right) - f\left(g\left(\dfrac{5}{3}\right)\right)$

$= g\left(\left[\dfrac{5}{3}\right]\right) - f\left(\left|\dfrac{5}{3}\right|\right)$

$= g(1) - f\left(\dfrac{5}{3}\right)$

$= |1| - \left[\dfrac{5}{3}\right] = 1 - 1 = 0$

$(gof) - (fog)\left(-\dfrac{5}{3}\right)$

$= g\left(f\left(-\dfrac{5}{3}\right)\right) - f\left(g\left(-\dfrac{5}{3}\right)\right)$

$= g\left[-\dfrac{5}{3}\right] - \left(\left|-\dfrac{5}{3}\right|\right)$

$= g(-2) - f\left(\dfrac{5}{3}\right)$

$= |-2| - \left[\dfrac{5}{3}\right]$

$= 2 - 1 = 1$

Sol.4 (C)

$f(x) = \sin^2\dfrac{x}{2} + \cos^2\dfrac{x}{2} = 1\ x \in R$

$g(x) = \sec^2 x - \tan^2 x = 1$,

$x \in R - \left[x : x = (2n+1)\dfrac{\pi}{2},\ n \in Z\right]$

$\therefore\ f(x) = g(x)\ x \in R$

$- \left[x : x = (2n+1)\dfrac{\pi}{2},\ n \in Z\right]$

Sol.5 (D)

$f(x) = \cos^2 x + \cos\left(\dfrac{\pi}{3} + x\right)$

$\left[\cos\left(\dfrac{\pi}{3} + x\right) - \cos x\right]$

$= \cos^2 x + \left[\dfrac{1}{2}\cos x - \dfrac{\sqrt{3}}{2}\sin x\right]$

$\left[\dfrac{1}{2}\cos x - \dfrac{\sqrt{3}}{2}\sin x - \cos x\right]$

$= \cos^2 x + \left[\dfrac{1}{2}\cos x - \dfrac{\sqrt{3}}{2}\sin x\right]$

$$\left[-\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x\right]$$

$$= \cos^2 x - \left[\frac{1}{4}\cos^2 x - \frac{3}{4}\sin^2 x\right]$$

$= 4\cos^2 x - \cos^2 x + 3\sin^2 x$

$= 3\cos^2 x + 3\sin^2 x$

$= 3(1) = 3 =$ constant.

A constant function is not periodic

$\therefore$ f(x) is not periodic

Sol.6 (D)

Since f(x) is defined for +ve as well as –ve value of x and $f(x) \geq 0 \ \forall \ x \in D_f$.

Hence the graph of f(x) will lie on right as well as on left of y-axis and above the x-axis.

$\therefore$ graph of f(x) lies in Ist and IInd quadrant.

Sol.7 (C)

Since $f(x) = \cos \sqrt{p}\, x$

$\therefore$ period of $f(x) = \dfrac{2\pi}{\sqrt{p}}$

$\Rightarrow \dfrac{2\pi}{\sqrt{p}} = \pi$ (given) $\Rightarrow p = 4$

$\Rightarrow [a] = 4 \Rightarrow a \in [4, 5)$

Sol.8 (C)

f(x) is defined when $\dfrac{x}{1-|x|} \geq 0$

If $x \geq 0$, then $1 - |x| > 0$

$\Rightarrow |x| < 1 \quad \Rightarrow x < 1$

$\therefore \quad x \in [0, 1)$

If $x \leq 0$, then $1 - |x| < 0$

$\Rightarrow |x| > 1 \qquad \Rightarrow x \in (-\infty, -1)$

$\therefore \quad D_f$ is $(-\infty, -1) \cup [0, 1)$.