Dear student following is an Easy level [O ● O O O] test paper. Score of 15 Marks in 10 Minutes would be a satisfactory performance. Questions 1-8 (+3, –1). (All questions have one option correct)

Q.1 Let f be a function such that $f(1) + 2f(2) + 3f(3) + ... n f(n) = n(n + 1) f(n)$ $n \geq 2$ and $f(1) = 1$, then the value of f(2005) is-

(A) $\frac{2}{2005}$ (B) $\frac{1}{4010}$

(C) $\frac{1}{8020}$ (D) $\frac{2}{2003}$

Q.2 If $f(x) = \frac{2005x + 153}{158x - 2005}$, then the least value of $f(f(x)) + f\left(f\left(\frac{1}{x}\right)\right)$ where $x > 0$ is $\left(x \neq \frac{2005}{158}\right)$

(A) 2 (B) 4

(C) 1 (D) Does not exist

Q.3 In a class of 55 students, the number of students studying different subjects are 23 in mathematics, 24 in physics, 19 in chemistry, 12 in mathematics and physics, 9 in mathematics and chemistry, 7 in physics and chemistry and 4 in all the three subjects. The number of students who have taken exactly one subject is-

(A) 6 (B) 9

(C) 7 (D) All of these

Q.4 Let $A = [x : x \in R, |x| < 1]$; $B = [x : x \in R, |x - 1| \geq 1]$ and $A \cup B = R - D$, then the set D is-

(A) $[x : 1 < x \leq 2]$ (B) $[x : 1 \leq x < 2]$

(C) $[x : 1 \leq x \leq 2]$ (D) None of these

Q.5 Let A = {a, b, c, d}, B = {b, c, d, e}. Then $O[(A \times B) \cap (B \times A)]$ is equal to-

(A) 3 (B) 6

(C) 9 (D) None of these

Q.6 Let $A = \{(x, y) : y = e^x, x \in R\}$
$B = \{(x, y) : y = e^{-x}, x \in R\}$. Then

(A) $A \cap B = \phi$ (B) $A \cap B \neq \phi$

(C) $A \cup B = R^2$ (D) None of these

Q.7 Domain of $f(x) = \sqrt{\sin^{-1}(2x) + \pi / 6}$ is-

(A) $\left[\frac{1}{4}, \frac{1}{2}\right]$ (B) $\left[\frac{-1}{4}, \frac{1}{2}\right]$

(C) $\left[\frac{1}{4}, \frac{1}{3}\right]$ (D) $\left[-\frac{1}{4}, \frac{1}{3}\right]$

Q.8 The function $f(x) = \sec [\log (x + \sqrt{1 + x^2})]$ is-

(A) Even (B) Odd

(C) Constant (D) None of these

MATHEMATICS IIT JEE (08 / 06 / 2007) (SETS, RELATIONS & FUNCTIONS) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	⬭	⬭	⬭	⬭	4	⬭	⬭	⬭	⬭	7	⬭	⬭	⬭	⬭
2	⬭	⬭	⬭	⬭	5	⬭	⬭	⬭	⬭	8	⬭	⬭	⬭	⬭
3	⬭	⬭	⬭	⬭	6	⬭	⬭	⬭	⬭					

ANSWER KEY

Que.	1	2	3	4	5	6	7	8
Ans.	B	A	D	B	C	B	B	A

SOLUTIONS

Sol.1 (B)

Given $f(1) + 2f(2) + 3f(3) + ... + nf(n)$

$= n(n + 1) f(n)$ ....(1)

$n \to n + 1$

$\Rightarrow f(1) + 2f(2) + ... + (n + 1) f(n + 1)$

$= (n + 1) (n + 2) f(n+ 1)$ ....(2)

(2) – (1)

$\Rightarrow (n + 1) f(n + 1)$

$=(n + 1) (n + 2) f(n + 1)–n(n + 1) f(n)$

$\Rightarrow n(n + 1) f(n) = (n + 1)^2 f(n + 1)$

$\Rightarrow nf(n) = (n + 1) f(n + 1)$

$\therefore 2f(2) = 3f(3) = ... = n f(n)$

$\therefore$ equation (1) reduces to

$f(1) + n(n - 1) f(n) = n(n + 1) f(n)$

$\Rightarrow f(1) = 2n f(n)$

or $f(n) = \dfrac{f(1)}{2n}$

$\therefore f(2005) = \dfrac{1}{2 \times 2005} = \dfrac{1}{4010}$

Sol.2 (A)

$f(x) = \dfrac{2005x + 153}{158x - 2005}$ ....(1)

Solving for x in terms of f(x),

$158x f(x) - 2005 f(x) = 2005x + 153$

$\Rightarrow x\{158f(x) - 2005\} = 153 + 2005 f(x)$

$\Rightarrow x = \dfrac{2005 f(x) + 153}{158 f(x) - 2005}$ ....(2)

Writing f(x) in place of x in (1) we get

$f(f(x)) = \dfrac{2005 f(x) + 153}{158 f(x) - 2005}$ ....(3)

(2) and (3) $\Rightarrow f(f(x)) = x$

Now, $f(f(x)) + f\left(f\left(\dfrac{1}{x}\right)\right) = x + \dfrac{1}{x} \ge 2\sqrt{x + \dfrac{1}{x}}$

(By AM – GM theorem)

$\therefore f(f(x)) + f\left(f\left(\dfrac{1}{x}\right)\right) \ge 2$

(Note that $x > 0$, so A.M. – G.M. can be applied)

Sol.3 (D)

$n(M) = 23, n(P) = 24, n(C) = 19$

$n(M \cap P) = 12, n(M \cap C) = 9,$

$n(P \cap C) = 7$

$n(M \cap P \cap C) = 4$

We have to find

$n(M \cap P' \cap C'), n(P \cap M' \cap C'),$

$n(C \cap M' \cap P')$

Now $n(M \cap P' \cap C')$

$= n[M \cap (P \cup C)']$

$= n(M) - n(M \cap (P \cup C))$

$= n(M) - n[(M \cap P) \cup (M \cap C)]$

$= n(M) - n(M \cap P) - n(M \cap C)$

$+ n(M \cap P \cap C)$

$= 23 - 12 - 9 + 4 = 27 - 21 = 6$

$n(P \cap M' \cap C') = n[P \cap (M \cup C)']$

$= n(P) - n[P \cap (M \cup C)]$

$= n(P) - n[(P \cap M) \cup (P \cap C)]$

$= n(P) - n(P \cap M) - n(P \cap C)$

$+ n(P \cap M \cap C)$

$= 24 - 12 - 7 + 4 = 9$

$n(C \cap M' \cap P') = n(C) - n(C \cap P)$

$- n(C \cap M) + n(C \cap P \cap M)$

$= 19 - 7 - 9 + 4 = 23 - 16 = 7$

Sol.4 (B)

$A = [x : x \in R, - 1 < x < 1]$

$B = [x : x \in R : x - 1 \le - 1 \text{ or } x - 1 \ge 1]$

$= [x : x \in R : x \le 0 \text{ or } x \ge 2]$

$\therefore A \cup B = R - D$

Where $D = [x : x \in R, 1 \le x < 2]$

Sol.5 (C)

Since $n(A \cap B) = 3$

$\therefore n((A \times B) \cap (B \times A)) = 3^2 = 9$

Sol.6 (B)

$A \cap B \neq \phi$

[$\because$ $y=e^x$, $y=e^{-x}$ will meet when $e^x = e^{-x}$

$\Rightarrow e^{2x} = 1 \quad \therefore x = 0 \qquad \therefore y = 1$

$\therefore$ A and B meet on (0, 1)]

Sol.7 (B)

For f(x) to exist, we must have

$\sin^{-1}(2x) + \frac{\pi}{6} \geq 0$ or, $\sin^{-1} 2x \geq -\frac{\pi}{6}$

Since $-\frac{\pi}{2} \leq \sin^{-1}(2x) \leq \frac{\pi}{2}$

So, $-\frac{\pi}{6} \leq \sin^{-1}(2x) \leq \frac{\pi}{2}$

or $\sin\left(-\frac{\pi}{6}\right) \leq 2x \leq \sin\left(\frac{\pi}{2}\right)$

$\Rightarrow -\frac{1}{2} \leq 2x \leq 1$

$\Rightarrow -\frac{1}{4} \leq x \leq \frac{1}{2}$

$\therefore$ Domain $= \left[-\frac{1}{4}, \frac{1}{2}\right]$

Sol.8 (A)

Since the function sec x is an even function and $\log\left(x + \sqrt{1+x^2}\right)$ is an odd function, therefore the function $\sec\left[\log\left(x + \sqrt{1+x^2}\right)\right]$ is an even function.