Dear student following is a Moderate level [O O ● O O] test paper. Score of 12 Marks in 10 Minutes would be a satisfactory performance. Questions 1-8 (+3, –1). (All questions have one option correct)

Q.1 If the function $f(x) = \sin^{-1}(x)$ and $g(x) = \sqrt{\sec^{-1} x}$ then domain of the fog(x) is-

(A) [0, 1] (B) [1, 2]

(C) [1, sec 1] (D) $[1, \infty)$

Q.2 If $f(x) = \sin([\pi]x) + \sin([\pi x])$, where [] is greatest integer function then $f\left(\frac{\pi}{2}\right)$ =

(A) sin 4 (B) sin 4 – 1

(C) sin 4 + 1 (D) cos 4

Q.3 A real valued function f(x) satisfies the functional equation

$f(x - y) = f(x) f(y) - f(a - x) f(a + y)$

where a is a given constant and f(0) = 1, f(2a – x) is equal to-

(A) – f(x) (B) f(x)

(C) f(a) + f(a – x) (D) f(–x)

Q.4 Let A = {x, y, z}, B = {u, v, w} and f : A $\rightarrow$ B be defined by f(x) = u, f(y) = v, f(z) = v. Then f is :

(A) Surjective but not injective

(B) Injective but not surjective

(C) Bijective

(D) None of them

Q.5 The function f : R $\rightarrow$ R defined by : f(x) = (x – 1) (x – 2) (x – 3) is-

(A) Onto but not one-one

(B) One-one but not onto

(C) Both one-one and onto

(D) None of them

Q.6 The domain of $f(x) - \sqrt{[\Delta(x)]}$, where

$$\Delta x = \begin{vmatrix} 1 & 1 & 1 \\ x & 3 & 4 \\ x^2 & 9 & 16 \end{vmatrix}^{-1}$$ is-

(A) (3, 4) (B) [3, 4]

(C) $(-\infty, 3) \cup (4, \infty)$

(D) $(-\infty, 3] \cup [4, \infty)$

Q.7 If $f(x) = (a - x^n)^{1/n}$, where a > 0 and n is a positive integer, then f[f(x)] is equal to-

(A) x (B) $x^{1/n}$

(C) x^n (D) $(a - x^n)$

Q.8 Which of the following functions is not an injective map ?

(A) $g(x) = |x + 1|, x \in [-1, \infty)$

(B) $g(x) = x + 1/x, x \in (0, \infty)$

(C) $h(x) = x^2 + 4x - 5, x \in (0, \infty)$

(D) $k(x) = e^{-x}, x \in [0, \infty)$

MATHEMATICS IIT JEE (09 / 06 / 2007) (SETS, RELATIONS & FUNCTIONS) ANSWER KEY

Name : ..... Roll No. : .....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○					

ANSWER KEY

Que.	1	2	3	4	5	6	7	8
Ans.	C	B	A	D	A	C	A	B

SOLUTIONS

Sol.1 (C)

We must have $0 \le \sqrt{\sec^{-1} x} \le 1$

$\Rightarrow \quad x \in [1, \sec 1]$

Sol.2 (B)

$$\sin\left([\pi]\frac{\pi}{2}\right) + \sin\left(\left[\pi\frac{\pi}{2}\right]\right)$$

$$= \sin\left(\frac{3\pi}{2}\right) + \sin\left[\frac{22\times 11}{7\times 7}\right]$$

$$= \sin\frac{3\pi}{2} + \sin 4 = \sin(4) - 1$$

Sol.3 (A)

$f(2a - x) = f(a - (x - a))$

$= f(a)\, f(x - a) - f(0)\, f(x)$

$= f(a)\, f(x - a) - f(x)$

$= - f(x)$

$[\because x = 0, y = 0, f(0) = f^2(0) - f^2(a)$

$\Rightarrow \quad f^2(a) = 0 \Rightarrow f(a) = 0]$

$\Rightarrow \quad f(2a - x) = - f(x)$

Sol.4 (D)

$f = \{(x, u), (y, v), (z, v)\}$ is the given mapping from $A = \{x, y, z\}$ to $B = \{u, v, w\}$

Clearly $f(y) = f(z) = v$, so f is many one and range of $f = \{u, v\} \subseteq B$, so it is into.

Thus f is many one into.

Sol.5 (A)

We have, $f(x) = (x - 1)(x - 2)(x - 3)$

$\therefore \quad f(1) = 0 = f(2) = f(3)$

Thus f is not one-one for each

$y \in R, \exists\ x \in R$

such that $f(x) = y$

$\therefore \quad f(x)$ is onto.

Hence f is onto but not one-one.

Sol.6 (C)

$$\Delta(x) = \begin{vmatrix} 1 & 1 & 1 \\ x & 3 & 4 \\ x^2 & 9 & 16 \end{vmatrix}$$

$= (x - 3)(3 - 4)(4 - x)$

$= (x - 3)(x - 4).$

$\Delta(x) > 0$ when $-\infty < x < 3$ or $4 < x < \infty$

Sol.7 (A)

$f(x) = (a - x^n)^{1/n},$

$f[f(x)] = f[(a - x^n)^{1/n}]$

$= [a - \{(a - x^n)^{1/n}\}^n]^{1/n}$

$= [a - (a - x^n)]^{1/n}$

$= (x^n)^{1/n} = x$

Sol.8 (B)

For choice (A), we have $f(x) = f(y)$;

$x, y \in [-1, \infty)$

$\Rightarrow \quad |x + 1| = |y + 1| \Rightarrow x + 1 = y + 1$

$\Rightarrow \quad x = y.$

Therefore f is an injection. For choice (B). $g(2) = 5/2$ and $g(1/2) = 5/2$. therefore $2 \neq ½$ but $g(2) = g(1/2)$. Thus, g(x) is not injective. It can be easily seen that choices (C) and (D) are also correct.