Q.1 Suppose $A_1, A_2, ... A_{30}$ are thirty sets each having 5 elements and $B_1, B_2, ... B_n$ are n sets each with 3 elements, let $\bigcup_{i=1}^{30} A_i = \bigcup_{j=1}^{n} B_j$ = S and each element of S belongs to exactly 10 of the A_i's and exactly 9 of the B_j's. Then n is equal to

(A) 15 (B) 3
(C) 45 (D) None of these

Q.2 Let S be the set of integers. For a, b $\in$ S, a R b if and only if $|a - b| < 1$, then

(A) R is not reflexive
(B) R is not symmetric
(C) R = $\{(a, a); a \in I\}$
(D) R is not an equivalence relation.

Q.3 Which of the following statements is true ?

(A) $P(A) \cap P(B) = P(A \cap B)$
(B) $P(A) \cup (B) = P(A \cup B)$
(C) $P(A \sim B) = P(A) \sim P(B)$
(D) None of these

Q.4 Given the relation R = {(1, 2), (2, 3)} on the set A = {1, 2, 3} the minimum number of ordered pairs which when added to R make it an equivalence relation is :

(A) 8 (B) 7
(C) 6 (D) 4

Q.5 Let R be a relation on the set N defined $\{(x, y)\ x, y \in N, 2x + y = 41\}$. Then R is :

(A) reflexive (B) symmetric
(C) transitive (D) None of the above

Q.6 If $X = \{8^n - 7n - 1 : n \in N\}$ and $Y = \{49(n - 1) : n \in N\}$, then

(A) $X \subseteq Y$ (B) $Y \subseteq X$
(C) $X = Y$ (D) None of these

Q.7 Out of 800 boys is a school, 224 played cricket, 240 played hockey and 336 played basketball. Of the total, 64 played both basketball and hockey; 80 played circket and basketball and 40 played cricket and hockey; 24 played all the three games. The number of boys who did not play any game is

(A) 160 (B) 240
(C) 216 (D) 128

Q.8 If R and R' are symmetric relations (not disjoint) on a set A, then the relation $R \cap R'$ is

(A) reflexive (B) symmetric
(C) transitive (D) None of these

Q.9 With reference to a universal set, the inclusion of a subset in another, is relation which is

(A) symmetric only (B) equivalence
(C) reflexive only (D) None of these

MATHEMATICS IIT JEE (CLASS TEST - 2) (SETS, RELATIONS) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9
Ans.	C	C	A	B	D	A	A	B	C

SOLUTIONS

Sol.1 (C)

$S = \bigcup_{i=1}^{30} A_i$, so $n(S) = \frac{1}{10}(5 \times 3) = 15$ (since element in the union S belongs to exactly 10 of the A_i's). Again $S = \bigcup_{j=1}^{n} B_j$ so

$$n(S) = \frac{1}{9}(3 \times n) = \frac{n}{3}.$$

Thus $\frac{n}{3} = 15$

$\Rightarrow$ n = 45.

Sol.2 (C)

For any integers a, b, $|a - b| < 1$ if and only if $|a - b| = 0$ so a = b.

Hence R = {(a, a); $a \in I$}. Thus R is reflexive, symmetric and transitive.

Sol.3 (A)

Since $A \cap B \subset A$, $A \cap B \subset B$ so $P(A \cap B) \subset P(A)$ and $P(A \cap B) \subset P(B)$. Thus $P(A \cap B) \subset P(A) \cap P(B)$. If $C \in P(A) \cap P(B)$ then $C \subset A$ and $C \subset B$ so $C \subset A \cap B$

$\Rightarrow$ $C \in P(A \cap B)$.

Hence $P(A) \cap P(B) = P(A \cap B)$.

Let A = {1, 2} and B = {2, 3} then $A \cup B$ = {1, 2, 3}, so P(A) = {ϕ, {1}, {2}, {1, 2}}; P(B) = {ϕ, {2}, {3}, {2, 3}};

$P(A \cup B)$ = {ϕ, {1}, {2}, {3}, {1, 2}, {1, 3} {2, 3}, {1, 2, 3}};

But $P(A) \cup P(B)$ = {ϕ, {1}, {2}, {3}, {1, 2}, {1, 3}} $\neq P(A \cup B)$.

Also A ~ B = {1} and P(A ~ B) = {ϕ, {1}} P(A) ~ P(B) = {ϕ, {1}, {1, 2}}.

Thus P(A ~ B) $\neq$ P(A) ~ P(B).

Sol.4 (B)

R is reflexive if it contains (1, 1), (2, 2), (3, 3)

$\because$ $(1, 2) \in R$, $(2, 3) \in R$

$\therefore$ R is symmetric if $(2, 1), (3, 2) \in R$

Now R = {(1, 1), (2, 2), (3, 3), (2, 1), (3, 2), (2, 3), (1, 2)} R wil be transitive if (3, 1), (1, 3) $\in$ R. Thus R becomes an equivalence relation by adding (1, 1) (2, 2), (3, 3), (2, 1), (3, 2), (1, 3), (3, 1). Hence the total number of ordered pairs is 7.

Sol.5 (D)

Here R = {(1, 39), (2, 37), (3, 35), (4, 33), (5, 31), (6, 29), (7, 27), (8, 25), (9, 23), (10, 21), (11, 19), (12, 17), (13, 15), (14, 13), (15, 11), (16, 9), (17, 7), (18, 5), (19, 3), (20, 1)}

Since $(1, 39) \in R$ but $(39, 1) \notin R \Rightarrow$ R is not symmetric. R is not transitive since (15, 11) $\in$ R and (11, 19) $\in$ R but (15, 19) $\notin$ R. Clearly R is not reflexive.

Sol.6 (A)

Since $8^n - 7n - 1 = (7 + 1)^n - 7n - 1$

$= 7^n + {}^nC_1\, 7^{n-1} + {}^nC_2\, 7^{n-2} + \ldots\ldots$

${}^nC_{n-1}\, 7 + {}^nC_n - 7n - 1$

$= {}^nC_2\, 7^2 + {}^nC_3\, 7^3 + \ldots\ldots\ {}^nC_n\, 7^n$

$({}^nC_0 = {}^nC_n,\ {}^nC_1 = {}^nC_{n-1}$ etc.)

$= 49[{}^nC_2 + {}^nC_3(7) + \ldots\ldots\ {}^nC_n\, 7^{n-2}]$

$\therefore$ $8^n - 7n - 1$ is a multiple of 49 for $n \geq 2$.

For n = 1, $8^n - 7n - 1 = 8 - 7 = 0$

For n = 2, $8^n - 7n - 1 = 64 - 14 - 1 = 49$

$\therefore$ $8^n - 7n - 1$ is a multiple of 49 for all $n \in N$.

Sol.7 (A)

n(C) = 224, n(H) = 240, n(B) = 336

$n(H \cap B) = 64$, $n(B \cap C) = 80$

$n(H \cap C) = 0$, $n(C \cap H \cap B) = 24$

$n(C^c \cap H^c \cap B^c) = n[(C \cup H \cup B)^c]$

$= n(U) - n(C \cup H \cup B)$

$= 800 - [n(C) + n(H) + n(B) - n(H \cap C) - n(H \cap B) - n(C \cap B) + n(C \cap H \cap B)]$

= 800 – [224 + 240 + 336 – 64 – 80 – 40 + 24]

= 800 – [824 – 184]

= 984 – 824 = 160.

Sol.8 (B)

Since R and R' are not disjoint, there is at least one ordered pair, say, (a, b) in $R \cap R'$.

But $(a, b) \in R'$

$\Rightarrow$ $(a, b) \in R$ and $(a, b) \in R'$

Since R and R' are symmetric relations,

we get

$(b, a) \in R$ and $(b, a) \in R'$

and consequently (b, a) $R \cap R'$.

Similarly, if any other ordered pair

$(c, d) \in R \cap R'$, then we must also have $(d, c) \in R \cap R'$.

Hence $R \cap R'$ is symmetric.

Sol.9 (C)

Let the universal set be $U = \{x_1, x_2, x_3 \ldots x_n\}$ We know every set is a subset of itself. Therefore, inclusion of a subset is reflexive. Now the elements of the set $\{x_1\}$ are included in the set $\{x_1, x_2\}$ but converse is not true i.e. $\{x_1\} \subset \{x_1, x_2\}$ but $\{x_1, x_2\} \not\subset \{x_1\}$

Hence, the inclusion of a subset is not symmetric.

Thus, the inclusion of a subset is not an equivalence relation.