Dear student following is a Moderate level [O ● O] test paper. Score of 15 Marks in 10 Minutes would be a satisfactory performance. Questions 1-8(+3, –1) (All questions have only one option correct)

Q.1 There are three coplanar parallel lines. If any p points are taken on each of the lines, the maximum number of triangles with vertices at these points is-

(A) $3p^2 (p - 1) + 1$ (B) $3p^2 (p - 1)$

(C) $p^2 (4p - 3)$ (D) None of these

Q.2 A teacher takes 3 children from her class to the zoo at a time as often as she can, but she does not take the same three children to the zero more than once. She finds that she goes to the zoo 84 times more than a particular child goes to the zoo. The number of children in her class is-

(A) 12 (B) 10

(C) 60 (D) None of these

Q.3 Two teams are to play a series of 5 matches between them. A match ends in a win or loss or draw for a team. A number of people forecast the result of each match and no two people make the same forecast for the series of matches. The smallest group of people in which one person forecasts correctly for all the matches will contain n people, where n is-

(A) 81 (B) 243

(C) 486 (D) None of these

Q.4 The letters of the word RACHIT are arranged in all possible ways and these words are written as in dictionary, then the rank of the word RACHIT is-

(A) 365 (B) 481

(C) 720 (D) None of these

Q.5 A student is allowed to select at the most n books out of (2n + 1). If the total number of ways by which he can select books is 63, then n equals-

(A) 5 (B) 7

(C) 3 (D) None of these

Q.6 20 persons are to be sitted around a circular table out of these 20 two are brothers. Then number of arrangement in which their will be exactly three persons between two brothers is -

(A) ${}^{19}C_3 \times 16! \times 5!$ (B) $\frac{19!}{3! \times 2!}$

(C) ${}^{18}C_3 \times 3! \times 2! \times 15!$ (D) None of these

Q.7 Let S be the set of all functions from the set A to the set A. If n(A) = k, then n(S) is-

(A) k! (B) k^k (C) $2^k - 1$ (D) 2^k

Q.8 Let A be the set of 4-digit numbers $a_1a_2a_3a_4$ where $a_1 > a_2 > a_3 > a_4$, then n(A) is equal to-

(A) 126 (B) 84

(C) 210 (D) None of these

MATHEMATICS IIT JEE (SEPT. 1st WEEK CLASS TEST 1) (PERMUTATION & COMBINATION) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	⬭	⬭	⬭	⬭	4	⬭	⬭	⬭	⬭	7	⬭	⬭	⬭	⬭
2	⬭	⬭	⬭	⬭	5	⬭	⬭	⬭	⬭	8	⬭	⬭	⬭	⬭
3	⬭	⬭	⬭	⬭	6	⬭	⬭	⬭	⬭					

ANSWER KEY

Que.	1	2	3	4	5	6	7	8
Ans.	C	B	B	B	C	C	B	C

SOLUTIONS

Sol.1 (C)

The number of triangles with vertices on different lines

$= {}^pC_1 \times {}^pC_1 \times {}^pC_1 = p^3$.

The number of triangles with two vertices on one line and the third vertex on any one of the other two lines

$= {}^3C_1 \{{}^pC_2 \times {}^{2p}C_1\} = 6p.\frac{p(p-1)}{2}$

so, the required number of triangles

$= p^3 + 3p^2 (p - 1) = p^2 (4p - 3)$.

Sol.2 (B)

The number of times the teacher goes to the zoo $= {}^nC_3$.

The number of times a particular child goes to the zoo $= {}^{n-1}C_2$.

From the question, ${}^nC_3 - {}^{n-1}C_2 = 84$.

or $(n - 1)(n - 2)(n - 3)$

$= 6 \times 84 = 9 \times 8 \times 7$

$\Rightarrow \quad n - 1 = 9 \quad \Rightarrow n = 10$

Sol.3 (B)

The smallest number of people = total number of possible forecasts = total number of possible results

$= 3 \times 3 \times 3 \times 3 \times 3 = 243$.

Sol.4 (B)

Keeping A at first place other letters can be arranged in 5! ways.

$\Rightarrow$ No. of words starting from

A = 1 × 5! = 120

Similarly No. of words starting from

C = 1 × 5! = 120

No. of words starting from

H = 1 × 5! = 120

No. of words starting from

I = 1 × 5! = 120

All these words will come before the words starting from R and RACHIT will be the first word starting from R.

Thus rank of word RACHIT = 4 × 120 + 1

= 481

Sol.5 (C)

Total number of ways of selecting one or more books upto n out of 2n + 1 books is given by

$63 = {}^{2n+1}C_1 + {}^{2n+1}C_2 + \ldots + {}^{2n+1}C_n$ .... (1)

or $63 = {}^{2n+1}C_{2n} + {}^{2n+1}C_{2n-1} \ldots\ldots + {}^{2n+1}C_{n+1}$

By ${}^nC_n = {}^nC_{n-r}$

$= {}^{2n+1}C_{n+1} + \ldots\ldots + {}^{2n+1}C_{2n-1} + {}^{2n+1}C_{2n}$ .... (2)

(On reversing the order)

On adding (1) and (2)

$126 = {}^{2n+1}C_1 + {}^{2n+1}C_2 + \ldots\ldots + {}^{2n+1}C_n + {}^{2n+1}C_{n+1} + \ldots\ldots + {}^{2n+1}C_{2n}$

$= 2^{2n+1} - {}^{2n+1}C_0 - {}^{2n+1}C_{2n+1}$

$= 2^{2n+1} - 1 - 1$

$\Rightarrow \quad 128 = 2^{2n+1} \quad \Rightarrow \quad n = 3$

Sol.6 (C)

At first we select the two brothers and then we select 3 persons out of remaining 18 by ${}^{18}C_3$ ways and form a group by keeping these three between 2 brothers and assume this group as a single person.

Now we have 15 + 1 = 16 persons and these can be arranged around a circular table in (16 – 1)! = 15! ways, and two brothers can be interchanged in 2! ways and three persons between them can be interchanged among themselves in 3! ways.

Thus total number of ways

$= {}^{18}C_3 \times 15! \times 2! \times 3!$

Sol.7 (B)

Each element of the set A can be given the image in the set A in k ways. So, the required number of functions,

i.e., $n(S) = k \times k \times \ldots\ldots\ldots$ (k times) $= k^k$.

Sol.8 (C)

Any selection of four digits from the ten digits 0, 1, 2, 3, ... , 9 gives one such number. So, the required number of numbers $= {}^{10}C_4 = 210$.