Dear student following is a tough level [O O ●] test paper. Score of 15 Marks in 15 Minutes would be a satisfactory performance. Questions 1-10(+3, –1) (All questions have only one option correct)

Q.1 Kunal Gaba has n objects, each of weight w. He weighs them in pairs and finds the sum of the weights of all possible pairs is 120. When his friend Rakshit weighs them in triplets, the sum of all possible weights is 240. The value of n is

(A) 7 (B) 6

(C) 5 (D) 10

Q.2 Let T_n denote the number of triangles which can be foremed by using vertices of a regular polygon of n sides. If $T_{n+1} - T_n = 21$, then n equals

(A) 5 (B) 7

(C) 6 (D) 4

Q.3 The number of ways of selecting 4 cards of an ordinary pack of playing cards so that exactly 3 of them are of the same denomination is

(A) 2496 (B) $^{13}C_3 \times {}^4C_3 \times 48$

(C) $^{52}C_3 \times 48$ (D) None of these

Q.4 A student is allowed to select at the most n books out of (2n + 1). If the total number of ways by which he can select books is 63, then n equals

(A) 5 (B) 7

(C) 3 (D) None of these

Q.5 In a certain test, there are n questions. In this test 2^{n-i} students gave wrong answers to at least i questions, where i = 1, 2,...n. If the total number of wrong answers given is 2047, then the value of n is

(A) 9 (B) 10

(C) 11 (D) 12

Q.6 The number of 5-digit numbers that can be made using the digits 1 and 2 in which at least one digit is different, is

(A) 30 (B) 31

(C) 32 (D) None of these

Q.7 How many numbers of 6 digits can be found from the digits of the number 112233 ?

(A) 30 (B) 60

(C) 90 (D) 120

Q.8 A question paper consists of two sections having 3 and 5 question respectively. The following note is given on the paper "It is not necessary to attempt all the question. One question from each section is comopulsory". In how many ways can a candidates select the questions ?

(A) 217 (B) 207

(C) 208 (D) 209

Q.9 The number of natural numbers which are less than 2.10^8 and which can be wirtten by means of the digits 1 and 2 is

(A) 772 (B) 870

(C) 900 (D) 766

Q.10 No. of permutations of 4 letters out of the letters of the word ARRANGEMENT are

(A) 1596 (B) 1956

(C) 1659 (D) 596

MATHEMATICS IIT JEE (SEPT.3rd WEEK CLASS TEST 4) (PERMUTATION & COMBINATION) ANSWER KEY

Name : ..... **Roll No. :** .....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○
										10	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	B	B	A	C	C	A	C	A	D	A

SOLUTIONS

Sol.1 (B)

According to the given condition

$({}^nC_2)\ (2w) = 120 \Rightarrow n(n-1)\,w = 120$

and $({}^nC_3)\ (3w) = 240 \Rightarrow n(n-1)(n-2)\,w = 480$

Thus, $\frac{n(n-1)(n-2)w}{n(n-1)w} = \frac{480}{120}$

$\Rightarrow n - 2 = 4 \qquad \Rightarrow n = 6$

Sol.2 (B)

The number of triangles that can be formed by using the vertices of a regular polygon is ${}^nC_3$ That is, $T_n = {}^nC_3$

Now, $T_{n+1} - T_n = 21$

$\Rightarrow {}^{n+1}C_3 - {}^nC_3 = 21$

$\Rightarrow {}^nC_2 + {}^nC_3 - {}^nC_3 = 21$

$[\because {}^{n+1}C_r = {}^nC_{r-1} + {}^nC_r]$

$\Rightarrow \frac{1}{2}\, n(n-1) = 21$

$\Rightarrow n = -6$ or 7.

As n is a positive integer, $n = 7$

Sol.3 (A)

We can chose one denomination in ${}^{13}C_1$ ways, then 3 cards of this denomination can be chosen in ${}^4C_3$ ways and one remaining cards can be chosen in ${}^{48}C_1$ ways. Thus, the total number of choices is $({}^{13}C_1)({}^4C_1)({}^{48}C_1) = 13 \times 4 \times 48 = 2496$.

Sol.4 (C)

Total number of ways of selecting one or more books upto n out of 2n + 1 books is given by

$63 = {}^{2n+1}C_1 + {}^{2n+1}C_2 + \ldots + {}^{2n+1}C_n \quad \ldots(1)$

or $63 = {}^{2n+1}C_{2n} + {}^{2n+1}C_{2n-1} \ldots + {}^{2n+1}C_{n+1}$

By ${}^nC_n = {}^nC_{n-r}$

$= {}^{2n+1}C_{n+1} + \ldots + {}^{2n+1}C_{2n-1} + {}^{2n+1}C_{2n} \quad \ldots(2)$

On reversing the order

On adding (1) and (2)

$126 = {}^{2n+1}C_1 + {}^{2n+1}C_2 + \ldots + {}^{2n+1}C_n + {}^{2n+1}C_{n+1} + \ldots + {}^{2n+1}C_{2n} = 2^{2n+1} - {}^{2n+1}C_0 - {}^{2n+1}C_{2n+1}$

$= 2^{2n+1} - 1 - 1$

$\Rightarrow 128 = 2^{2n+1} \qquad \Rightarrow n = 3.$

Sol.5 (C)

The number of students ansering exactly i($1 \le i \le n-1$) questions wrongly is

$2^{n-i} - 2^{n-i-1}$

The numbers of students answering all n questions wrongly is 2^0. Thus, the total number of wrong answers

$= 1(2^{n-1} - 2^{n-2}) + 2(2^{n-2} - 2^{n-3}) + \ldots\ldots$

$+ (n-1)(2^1 - 2^0) + n(2^0)$

$= 2^{n-1} + 2^{n-2} + 2^{n-3} + \ldots + 2^0 = 2^n - 1$

Thus, $2^n - 1 = 2047$

or $2^n = 2048 = 2^{11}$

$\Rightarrow n = 11.$

Sol.6 (A)

Total number of numbers without restriction = 2^5.

Two numbers have all the digits equal. So, the required number of numbers = $2^5 - 2$.

Sol.7 (C)

Required 6 digit numbers = $\frac{6!}{2!2!2!} = 90.$

Sol.8 (A)

Here we have two sections A and B (say), the section A has 3 questions and section B has 5 questions and one question from each section is compulsory according to the given direction.

$\therefore$ Number of ways selecting one or more than one question from section A is

$2^3 - 1 = 7$

and number of ways selecting one or more than one question from section B is

$2^5 - 1 = 31$

Hence, by the principle of multiplication, the required number of ways in which a candidate can select the questions = $7 \times 31 = 217$.

Sol.9 (D)

The required numbers are 1, 2, 11, 12, 21, 22, ..., 122222222.

Let us calculate how many numbers are these.

There are 2 one-digit such numbers. There are 2^2 two-digit such numbers and so on.

There are 2^8 eight-digit such numbers. All the digit numbers beginning with 1 and written by means of 1 and 2 are smaller than 2.10^8. Thus, there are 2^8 such nine digit numbers. Hence the required number of numbers is

$2 + 2^2 + 2^3 + ... + 2^8 + 2^8$

$= \dfrac{2(2^8 - 1)}{2 - 1} + 2^8$

$= 2^9 - 2 + 2^8$

$= 766$

Sol.10 (A)

This is 4! times the coeff. of x^4 is

$$\left(1 + x + \frac{x^2}{2!}\right)^4 (1 + x^3)$$

(As there are 2A's, 2R's, 2E's, 2N's, 1G, 1M and 1T

$= 4! \left(\dfrac{17}{2} + 30 + 24 + 4\right)$

$= 1596$