Dear student following is a tough level [○ ○ ●] test paper. Score of 15 Marks in 15 Minutes would be a satisfactory performance. Questions 1-10(+3, –1) (All questions have only one option correct)

Q.1 The number of ways in which we can post 5 letters in 10 letters boxes is

(A) 50 (B) 5^{10}

(C) 10^5 (D) None of these

Q.2 The number of ways of arranging letters of the world HAVANA so that V and N do not appear together is

(A) 40 (B) 60

(C) 80 (D) 100

Q.3 There are 15 points in a plane of which exactly 8 are collinear. Find the number of straight lines obtained by joining the points.

(A) 87 (B) 77

(C) 78 (D) 79

Q.4 Find the number of rectangles that you can find on chessboard.

(A) 1295 (B) 1296

(C) 1269 (D) 1276

Q.5 Find the number of ways in which we can choose 3 letters of the word SUDESH.

(A) 60 (B) 64

(C) 62 (D) 72

Q.6 Find the number of integers greater than 7000 that can be formed with the digits 3, 5, 7, 8 and 9, no digit being repeated.

(A) 192 (B) 144

(C) 120 (D) 5!

Q.7 The number of ways in which we can get a score of 11 by throwing three dice is

(A) 18 (B) 27

(C) 45 (D) 56

Q.8 The number of five digit numbers that can be formed by using digits 1, 2, 3 only, such that exactly three digit of the formed number are same is

(A) 30 (B) 60

(C) 90 (D) None of these

Passage

If 25 identical things be distributed among five persons, then

Q.9 The number of ways each receives at least one :

(A) ${}^{24}C_4 - 5$ (B) ${}^{24}C_4 - 5\ {}^{13}C_4$

(C) ${}^{24}C_4 - 5.\ {}^{13}C_4$ (D) ${}^{24}C_4$

Q.10 The number of ways each receives at least one thing but not more than eleven

(A) ${}^{24}C_4 - 5\ {}^{12}C_4$ (B) ${}^{24}C_4 - 5$

(C) ${}^{24}C_4 - 5\ {}^{13}C_4$ (D) None of these

MATHEMATICS IIT JEE (SEPT.3rd WEEK CLASS TEST 5) (PERMUTATION & COMBINATION) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○
										10	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	C	C	C	B	D	A	B	B	D	C

SOLUTIONS

Sol.1 (C)

We can post the first letter in 10 ways, the second leter in 10 ways and so on. Thus, the number of ways of posting 5 letters in 10 letter boxes is $10 \times 10 \times 10 \times 10 \times 10 = 10^5$

Sol.2 (C)

We can arrange the letters H, A, A, A in

$\frac{4!}{3!} = 4$ ways .

If one possible arrangement is

X X X X

Then we can V, N at any of the two places marked with O in the following arrangement.

O X O X O X O X O

This, we arrange V and N in

$= {}^5P_2 = 20$ ways

Thus, the number of ways in which letters can be arranged is $4 \times 20 = 80$.

Sol.3 (C)

For a line we require two points. Therefore, the number of lines which we can obtain is ${}^{15}C_2 = (15 \times 14)/2 = 105$. Since 8 of these points lie on a straight line, we loose ${}^8C_2 = 28$ lines and get just one line on which these points lie. Therefore, the number of lines is $105 - 28 + 1 = 78$

Sol.4 (B)

There are 9 horizontal and 9 vertical lines on a chessboard. To form a rectangle we require two horizontal and two vertical lines. Thus, the number of rectangles on the chessboard is $({}^9C_2)\ ({}^9C_2) = 36 \times 36 = 1296$

Sol.5 (D)

The letters of the word SUDESH are (S,S) U, D, E, H.

We can choose 3 distinct letters in ${}^5C_3$ ways and arrange them ${}^3P_3$ ways.

In this case the number of ways in $({}^5C_3)({}^3P_3) = (10)(6) = 60$.

We can choose a pair (S, S) in one ways and one more letter in 4 ways. We can arrange these letters in 3!/2! ways. In this case the number of ways is (4)(3) = 12.

Thus, there are 60 + 12 = 72 ways.

Sol.6 (A)

A five digit integers is always greater than 7000. The number of such integers is ${}^5P_5 = 5! = 120$. For a four digit integer to be greater than 7000, it must begin with 7, 8 or 9. The number of such integer is $(3)({}^4P_3) = 72$. Hence, the required number of such integers is 120 + 72 = 192.

Sol.7 (B)

Coefficient of x^{11} in $(x + x^2 + ... + x^6)^3$

$=$ coefficient of x^8 in $(1 - x^6)^3 (1 - x)^{-3}$

$=$ coefficient of x^8 in $(1 - 3x^6)(1 + {}^3C_1x + {}^4C_2x^2 + ...)$

$= {}^{10}C_8 - 3({}^4C_2) = 27$

Sol.8 (B)

$({}^3C_1)\ \frac{5!}{3!} = 60$

Sol.9 (D)

Let person P_i gets x_i number of things such that :

$$x_1 + x_2 + x_3 + x_4 + x_5 = 25$$

where $x_1, x_2, x_3, x_4, x_5 \geq 1$

$\therefore$ Let $a_1 = x_1 - 1 \geq 0$

$a_2 = x_2 - 1 \geq 0$

$a_3 = x_3 - 1 \geq 0$

$a_4 = x_4 - 1 \geq 0$

$a_5 = x_5 - 1 \geq 0$

$x_1 + x_2 + x_3 + x_4 + x_5 = 25$

$\Rightarrow (a_1 + 1) + (a_2 + 1) + (a_3 + 1)$

$+ (a_4 + 1) + (a_5 + 1) = 25$

$\Rightarrow a_1 + a_2 + a_3 + a_4 + a_5 = 20$

$\therefore$ Number of solutions are $^{20+5-1}C_{5-1} = {}^{24}C_4$

Sol.10 (C)

In this case, $1 \leq x_i \leq 11$

Required number of ways is equal to the coefficient of x^{25} in $(x + x^2 + ... + x^{11})^5$

$\Rightarrow$ Coefficient of x^{20} in $(1 + x + x^2 + ... + x^{10})^5$

$\Rightarrow$ Coefficient of x^{20} in $(1 - x^{11})^5 (1 - x)^{-5}$

$\Rightarrow$ Coefficient of x^{20} in $(1 - 5x^{11})(1 - x)^{-5}$

$\Rightarrow {}^{24}C_{20} - 5.\ {}^{13}C_9$

$\Rightarrow {}^{24}C_4 - 5.\ {}^{13}C_4$