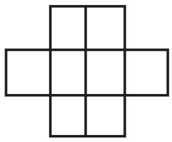
Dear student following is a Moderate level [○ ● ○] test paper. Score of 18 Marks in 15 Minutes would be a satisfactory performance. Questions 1-9(+3, –1) (All questions have only one option correct)

Q.1 Five letters are to be placed in five addressed envelopes. Then number of ways in which no letter goes to the envelope assigned to it is-

(A) 32 (B) 26
(C) 44 (D) None of these

Q.2 Six X_S have to be placed in the squares of the diagram given below such that each row contains at least one X. The number of ways in which this can be done is :



(A) 6 (B) 28
(C) 26 (D) None of these

Q.3 The number of ways in which 15 identical balls can be distributed among 5 children is -

(A) ${}^{19}C_4$ (B) ${}^{15}C_{11}$
(C) ${}^{15}C_5$ (D) None of these

Q.4 20 persons are to be sitted around a circular table out of these 20 two are brothers. Then number of arrangement in which their will be exactly three persons between two brothers is-

(A) ${}^{19}C_3 \times 16! \times 5!$ (B) $\frac{19!}{3! \times 2!}$
(C) ${}^{18}C_3 \times 3! \times 2! \times 15!$
(D) None of these

Q.5 If ${}^nC_4$, ${}^nC_5$ and ${}^nC_6$ are in A.P., then value of n can be-

(A) 6 (B) 7 (C) 8 (D) 9

Q.6 In a group of 8 girls, two girls are sisters. The number of ways in which the girls can sit so that two sisters are not sitting together is-

(A) 4820 (B) 1410
(C) 2830 (D) None of these

Q.7 If $a_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$, then $\sum_{r=0}^{n} \frac{r}{{}^nC_r}$ equals-

(A) $\frac{n}{2} a_n$ (B) $\frac{n}{4} a_n$
(C) na_n (D) $(n-1)a_n$

Q.8 The sum $S = \frac{1}{9!} + \frac{1}{3!\,7!} + \frac{1}{5!\,5!} + \frac{1}{7!\,3!} + \frac{1}{9!}$ equals-

(A) $\frac{2^9}{10!}$ (B) $\frac{2^{10}}{8}$
(C) $\frac{2^{11}}{9!}$ (D) None of these

Q.9 The value of

$$E = \frac{(1+17)\left(1+\frac{17}{2}\right)\left(1+\frac{17}{3}\right)......\left(1+\frac{17}{19}\right)}{(1+19)\left(1+\frac{19}{2}\right)\left(1+\frac{19}{3}\right).....\left(1+\frac{19}{17}\right)} \text{ is}$$

(A) 1 (B) ${}^{36}C_{17}$ (C) 2/19 (D) ${}^{36}C_{18}$

MATHEMATICS IIT JEE (AUGUST 4th WEEK CLASS TEST 2) (PERMUTATION & COMBINATION) ANSWER KEY

Name : .... **Roll No. :** ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9
Ans.	C	C	A	C	B	D	A	A	A

SOLUTIONS

Sol.1 (C)

By Dearrangement principle required number of ways

$$= 5!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}\right)$$

$$= 5! - 5! + \frac{5!}{2!} - \frac{5!}{3!} + \frac{5!}{4!} - 1$$

$= 60 - 20 + 5 - 1$

$= 44$

Sol.2 (C)

There are only two ways in which a row will remain empty i.e., if we place 4X in 2nd row and 2X in 3rd row or if 4X in 2nd row and 2X is Ist row.

Thus number of ways in which X_S can be filled, such that each row contains at least one X

= Total number of arrangements

– number of ways in which one row remain empty

$= {}^8C_6 - 2$

$$= \frac{8!}{2!\ 6!} - 2 = 28 - 2 = 26$$

Sol.3 (A)

Let x_1, x_2, x_3, x_4 and x_5 be the no. of balls given to the children then we have

$x_1 + x_2 + x_3 + x_4 + x_5 = 15$ ....(1)

Since negative and fractional no. of balls can not be given to any child then we have each $x_i \geq 0$ and x_i is an integer.

Thus number of ways of distributing the balls = Number of non negative integral solutions of eq. (1)

= Coeff. of t^{15} in the expansion of $(1 - t)^{-5}$

$= {}^{15+5-1}C_{15} = {}^{19}C_{15} = {}^{19}C_4$

Sol.4 (C)

At first we select the two brothers and then we select 3 persons out of remaining 18 by ${}^{18}C_3$ ways and form a group by keeping these three between 2 brothers and assume this group as a single person.

Now we have 15 + 1 = 16 persons and these can be arranged around a circular table in (16 – 1)! = 15! ways, and two brothers can be interchanged in 2! ways and three persons between them can be interchanged among themselves in 3! ways.

Thus total number of ways

$= {}^{18}C_3 \times 15! \times 2! \times 3!$

Sol.5 (B)

As ${}^nC_4$, ${}^nC_5$ and ${}^nC_6$ are in A.P.,

$2({}^nC_5) = {}^nC_4 + {}^nC_6$

$$\Rightarrow\quad 2 = \frac{{}^nC_4}{{}^nC_5} + \frac{{}^nC_6}{{}^nC_5}$$

$$= \frac{n!}{4!(n-4)!}\,\frac{5!(n-5)!}{n!} + \frac{n!}{6!(n-6)!}\,\frac{5!(n-5)!}{n!}$$

$$\Rightarrow\quad 2 = \frac{5}{n-4} + \frac{n-5}{6}$$

$\Rightarrow\quad n^2 - 21n + 98 = 0$

$\Rightarrow\quad n = 7, 14$

Sol.6 (D)

The required number of ways

= the number of ways in which 8 girls can sit - the number of ways in which two sisters are together

= 8! – (2) (7!) = 30240

Sol.7 (A)

$$\text{Let } b_n = \sum_{r=0}^{n}\frac{r}{{}^nC_r} = \sum_{r=0}^{n}\frac{n-r}{{}^nC_{n-r}} = \sum_{r=0}^{n}\frac{n-r}{{}^nC_r}$$

$$\Rightarrow\quad 2b_n = \sum_{r=0}^{n}\frac{r+(n-r)}{{}^nC_r} = n\sum_{r=0}^{n}\frac{1}{{}^nC_r} = na_n$$

$$\Rightarrow\quad b_n = \frac{n}{2}a_n$$

Sol.8 (A)

We can write S as follows

$$S = \frac{1}{10!}\,[{}^{10}C_1 + {}^{10}C_3 + {}^{10}C_5 + {}^{10}C_7 + {}^{10}C_9]$$

$$= \frac{1}{10!}\,(2^9)$$

Sol.9 (A)

We have

$$(1 + k)\left(1+\frac{k}{2}\right)\left(1+\frac{k}{3}\right) \ldots\ldots \left(1+\frac{k}{n}\right)$$

$$= \frac{(1+k)\,(2+k)\,(3+k)\ldots.(n+k)}{(2)\,(3)\ldots.(n)}$$

$$= \frac{(n+k)!}{k!\,n!}$$

$$= {}^{n+k}C_k$$

Thus, both numerator and denominator of E equals ${}^{36}C_{17} = {}^{36}C_{19}$

$\therefore$ E = 1