Dear student following is a Moderate level [O ● O] test paper. Score of 18 Marks in 15 Minutes would be a satisfactory performance. Questions 1-6(+3, –1), 7-8(+6, 0). (Questions may have more than one option correct)

Q.1 The degree of the remainder when $x^{2007} - 1$ is divided by $(x^2 + 1)(x^2 + x + 1)$ is-

(A) 3 (B) 2
(C) 1 (D) 0

Q.2 Let $f(x) = ax^2 + bx + c$, where $a, b, c \in R$. Suppose $|f(x)| \le 1 \ \forall x \in [0, 1]$ then

(A) $|a| \le 8$ (B) $|b| \le 8$
(C) $|c| \le 1$
(D) $|a| + |b| + |c| \le 17$

Q.3 If $ax^2 + bx + c$, $a, b, c \in R$ has no real zeros, and if $c < 0$ then,

(A) $a < 0$ (B) $a + b + c > 0$
(C) $a > 0$ (D) None of these

Q.4 The number of quadratic equations $ax^2 + bx + c = 0$ such that $ax^2 + bx + c + \lambda = 0$ has two distinct positive roots for each $\lambda \in R$, is

(A) 1 (B) 6
(C) Infinite (D) None of these

Q.5 The number of polynomials p(x) with integral coefficients satisfying the conditions $p(1) = 2$, $p(3) = 1$.

(A) 0 (B) 1
(C) 2 (D) 3

Q.6 If $\cot\theta$, $\operatorname{cosec}\theta$ are the roots of the equation $ax^2 + bx + c = 0$ and $\Delta = b^2 - 4ac$, then a^4 equals

(A) $b\Delta$ (B) $b^2\Delta$
(C) $bc\Delta$ (D) $c\Delta$

Q.7 Let α, β, γ be three numbers such that

$$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{1}{2},\ \frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} = \frac{9}{4},$$

and $\alpha + \beta + \gamma = 2$.

(i) $\alpha\beta\gamma$ (a) 6
(ii) $\beta\gamma + \gamma\alpha + \alpha\beta$ (b) 8
(iii) $\alpha^2 + \beta^2 + \gamma^2$ (c) –2
(iv) $\alpha^3 + \beta^3 + \gamma^3$ (D) –1

(A) (i)↔(c), (ii)↔(d), (iii)↔(a), (iv)↔(b)
(B) (i)↔(a), (ii)↔(d), (iii)↔(c), (iv)↔(b)
(C) (i)↔(c), (ii)↔(a), (iii)↔(d), (iv)↔(b)
(D) (i)↔(b), (ii)↔(d), (iii)↔(a), (iv)↔(c)

Q.8 The set of value(s) of $k \in R$ for which

(i) $kx^2 - (k + 1)x + 2k - 1 = 0$ has no real roots (a) $\{1, -2\}$
(ii) $x^2 - 2(4k - 1)x + 15k^2 - 2k - 7 > 0$ for each x (b) $(-\infty, -1/7) \cup (1, \infty)$
(iii) Sum of the roots of $x^2 + (2 - k - k^2)x - k^2 = 0$ is zero (c) $\{-4\}$
(iv) The roots of $x^2 + (2k - 1)x + k^2 + 2 = 0$ are in the ratio 1 : 2 (d) (2, 4)

(A) (i)↔(b), (ii)↔(d), (iii)↔(a), (iv)↔(c)
(B) (i)↔(d), (ii)↔(b), (iii)↔(a), (iv)↔(c)
(C) (i)↔(a), (ii)↔(d), (iii)↔(b), (iv)↔(c)
(D) (i)↔(c), (ii)↔(d), (iii)↔(a), (iv)↔(b)

MATHEMATICS IIT JEE (AUGUST 1st WEEK CLASS TEST 3) (QUADRATIC EQUATION) ANSWER KEY

Name : .... **Roll No. :** ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○					

ANSWER KEY

Que.	1	2	3	4	5	6	7	8
Ans.	A	All	A	D	A	B	A	A

SOLUTIONS

Sol.1 **(A)**

Write as

$x^{2007} - 1$

$= q(x)(x^2+1)(x^2+x+1) + ax^3 + bx^2 + cx + d$

and substitute $x = \omega, \omega^2$, i, –i to show a = 1.

Sol.2 **(All)**

Putting x = 0, 1, 1/2, we obtain

$|c| \le 1,\ |a+b+c| \le 1, \left|\frac{1}{4}a + \frac{1}{2}b + c\right| \le 1$

$\Rightarrow -1 \le c \le 1,\ -1 \le a+b+c \le 1,$

$-4 \le a + 2b + 4c \le 4$

$\Rightarrow -4 \le 4a + 4b + 4c \le 4$

and $-4 \le -a - 2b - 4c \le 4$

Adding we get

$-8 \le 3a + 2b \le 8$

Also $-8 \le a + 2b \le 8$

$\therefore\ -16 \le 2a \le 16 \Rightarrow |a| \le 8$

Since $-1 \le -c \le 1,\ -8 \le -a \le 8,$

We get $-16 \le 2b \le 16 \Rightarrow |b| \le 8$

Thus, $|a| + |b| + |c| \le 17.$

Sol.3 **(A)**

Let $f(x) = ax^2 + bx + c$.

Since f(x) has no real zeros,

either f(x) > 0 or f(x) < 0 for all $x \in R$.

Since f(0) = c < 0,

we get $f(x) < 0\ \forall\ x \in R$.

Therefore a < 0

as the parabola y = f(x) open downwards.

Sol.4 **(D)**

If a < 0 and $\lambda > -c$, then $ax^2 + bx + c + \lambda = 0$ has one negative root.

If a > 0, then for $\lambda > -c + b^2/a$ the equation has no real roots.

Sol.5 **(A)**

Let p(x) be one such polynomial.

Let $p_1(x)$, be such that

$p(x) - 1 = (x-3)\,p_1(x)$

$\Rightarrow p(1) - 1 = -2p_1(1)$

$1 = -2p_1(1)$

But this is not possible as both 2 and $p_1(1)$ are integers.

Sol.6 **(B)**

$\operatorname{cosec}\theta + \cot\theta = -b/a,\ \operatorname{cosec}\theta\cot\theta = c/a$

As $\operatorname{cosec}^2\theta - \cot^2\theta = 1$, we get

$\operatorname{cosec}\theta - \cot\theta = -\frac{a}{b}$

$\therefore \operatorname{cosec}\theta = -\frac{1}{2}\left(\frac{b}{a} + \frac{a}{b}\right)$

and $\cot\theta = \frac{1}{2}\left(\frac{a}{b} - \frac{b}{a}\right)$

Thus, $-\frac{1}{4}\left(\frac{a^2}{b^2} - \frac{b^2}{a^2}\right) = \frac{c}{a}$

$\Rightarrow a^4 - b^4 = -4acb^2$

$\Rightarrow a^4 = b^2(b^2 - 4ac)$

Sol.7 **(A)**

$$2\left(\frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha} + \frac{1}{\alpha\beta}\right) = \left(\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}\right)^2 - \left(\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}\right)$$

$\Rightarrow \frac{2(\alpha+\beta+\gamma)}{\alpha\beta\gamma} = -2 \Rightarrow \alpha\beta\gamma = -2$

Also, $\frac{\beta\gamma + \gamma\alpha + \alpha\beta}{\alpha\beta\gamma} = \frac{1}{2}$

$\Rightarrow \beta\gamma + \gamma\alpha + \alpha\beta = -1$

$\therefore \alpha^2 + \beta^2 + \gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\beta\gamma + \gamma\alpha + \alpha\beta)$
$= 6$

Lastly $\alpha^3 + \beta^3 + \gamma^3 - 3\alpha\beta\gamma$

$= (\alpha+\beta+\gamma)(\alpha^2 + \beta^2 + \gamma^2 - \beta\gamma - \gamma\alpha - \alpha\beta)$

Sol.8 **(A)**

(i) $(k+1)^2 - 4k(2k-1) < 0,\ k \ne 0$

$\Rightarrow 7k^2 - 6k - 1 > 0$

$\Rightarrow (7k+1)(k-1) > 0$

$\Rightarrow k < -1/7$ or $k > 1$

(ii) $(4k-1)^2 - (15k^2 - 2k - 7) < 0$

$\Rightarrow k^2 - 6k + 8 < 0 \Rightarrow (k-2)(k-4) < 0$

$\Rightarrow 2 < k < 4$

(iii) $2 - k - k^2 = 0 \Rightarrow k = 1, -2$

(iv) $\alpha + 2\alpha = -(2k-1),$

$\alpha(2\alpha) = k^2 + 2$

$\Rightarrow \frac{2}{9}(2k-1)^2 = k^2 + 2.$

$\Rightarrow (k+4)^2 = 0 \quad \Rightarrow k = -4$