*Dear student following is a Moderate level [O ● O] test paper. Score of 18 Marks in 15 Minutes would be a satisfactory performance. Questions 1-10(+3, –1), (All questions have only one option correct)*

**Q.1** If $x = 2 + 2^{2/3} + 2^{1/3}$, then the value of $x^3 - 6x^2 + 6x$ is

(A) 3 (B) 2

(C) 1 (D) None of these

**Q.2** If a, b, c are real and $x^3 - 3b^2x + 2c^3$ is divisible by x – a and x – b,then

(A) a = –b = –c (B) a = 2b = 2c

(C) a = b = c or a = –2b = –2c

(D) None of these

**Q.3** If a, b, c, d be four consecutive terms of an increasing A.P., then the roots of the equation $(x - a)(x - c) + 2(x - b)(x - d) = 0$ are

(A) Real and distinct (B) Complex

(C) Equal roots (D) None

**Q.4** The value of λ for which

$2x^2 - 2(2\lambda + 1)x + \lambda(\lambda + 1) = 0$ may have one root less than λ and other root greater than λ are given by

(A) $1 > \lambda > 0$ (B) $-1 < \lambda < 0$

(C) $\lambda \geq 0$ (D) $\lambda < 0$ or $\lambda < -1$

**Q.5** If the equation $x^3 - 3x + a = 0$ has distinct roots between 0 and 1 then a belongs to

(A) (–1, –2) (B) (0, 2)

(C) (2, 3) (D) None

**Q.6** The middle point of the interval in which

$x^2 + 2\left(\sqrt{x}\right)^2 - 3 \leq 0$ is

(A) 1/2 (B) 1

(C) 0 (D) –1/2

**Q.7** All the values of a for which the equation $2x^2 - 2(2a + 1)x + a(a + 1) = 0$ has two roots, one of which is greater than a and the other is smaller than a.

(A) $a \in (1, \infty)$ (B) $(-\infty, -1)$

(C) $(-\infty, -1) \cup (0, \infty)$ (D) $(-\infty, \infty)$

**Q.8** If $3x^2 - 2(a - d)x + (a^2 + 2(b^2 + c^2) + d^2) = 2(ab + bc + cd)$, then

(A) a, b, c, d are in G.P.

(B) a, b, c, d are in H.P.

(C) a, b, c, d are in A.P.

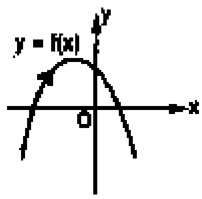
(D) None of these

**Q.9** If $|f(x) + 6 - x^2| = |4 - x^2| + 2 + |f(x)|$, then f(x) is necessarily non-negative in

(A) [–2, 2] (B) $(-\infty, -2) \cup (2, \infty)$

(C) $[-\sqrt{6}, \sqrt{6}]$ (D) None of these

**Q.10** Consider the graph of $f(x) = ax^2 + bx + c$ in the adjacent figure. We can conclude that




(A) a > 0, b > 0, c > 0

(B) a > 0, b < 0, c < 0

(C) a < 0, b < 0, c > 0

(D) a < 0, b < 0, c > 0 and a + b + c < 0

## MATHEMATICS IIT JEE ( AUGUST 1st WEEK CLASS TEST 4) (QUADRATIC EQUATION) ANSWER KEY

Name : .............................................. Roll No. : ..............................

| | A | B | C | D | | A | B | C | D | | A | B | C | D |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | ⬭ | ⬭ | ⬭ | ⬭ | 5 | ⬭ | ⬭ | ⬭ | ⬭ | 9 | ⬭ | ⬭ | ⬭ | ⬭ |
| 2 | ⬭ | ⬭ | ⬭ | ⬭ | 6 | ⬭ | ⬭ | ⬭ | ⬭ | 10 | ⬭ | ⬭ | ⬭ | ⬭ |
| 3 | ⬭ | ⬭ | ⬭ | ⬭ | 7 | ⬭ | ⬭ | ⬭ | ⬭ | | | | | |
| 4 | ⬭ | ⬭ | ⬭ | ⬭ | 8 | ⬭ | ⬭ | ⬭ | ⬭ | | | | | |

## ANSWER KEY

| Que. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Ans. | B | C | A | D | B | A | C | C | A | C |

# SOLUTIONS

**Sol.1 (B)**

$(x-2)^3 = 2^2 + 2 + 3.\ 2(x-2)$

or $x^3 - 6x^2 + 12x - 8 = 6 + 6x - 12$

$\therefore$ $x^3 - 6x^2 + 6x = 2$

**Sol.2 (C)**

$f(x) = x^3 - 3b^2x + 2c^3$

Since f(x) is divisible by x − a and x − b then a, b are roots of f(x) = 0

f(a) = 0, f(b) = 0

$a^3 - 3b^2a + 2c^3 = 0,\ b^3 - 3b^3 + 2c^3 = 0$

Second relation implies b = c and then from Ist, we get $a^2 - 3b^2a + 2b^3 = 0$

or $(a-b)(a^2 + ab - 2b^2) = 0$

When a − b = 0 then a = b = c ..... (1)

When $a^2 + ab - 2b^2 = 0$

or (a + 2b) (a − b) = 0

$\therefore$ a = −2b = −2c ....(2)

Both (1) and (2) $\Rightarrow$ option (c).

**Sol.3 (A)**

Let a, b, c, d be chosen as

$a,\ a+\lambda, a+2\lambda,\ a+3\lambda$ respectively

where $\lambda > 0$ as the A.P. is increasing.

The given equation is

$3x^2 - x(a + c + 2b + 2d) + ac + 2bd = 0$

or $3x^2 - x(6a + 10\lambda) + (3a^2 + 10a\lambda + 6\lambda^2) = 0$

$D = B^2 - 4AC = (6a + 10\lambda)^2$

$- 4.3\,(3a^2 + 10a\lambda + 6\lambda^2)$

$= (100 - 72)\lambda^2 = 28\lambda^2 = +$ ive

Hence the roots are real and distinct.

**Sol.4 (D)**

$f(x) = 2(x-\alpha)(x-\beta)$

Given that $\lambda$ lies between $\alpha$ and $\beta$.

Hence f($\lambda$)= −ive by $\delta$ 7P. 135.

Now f($\lambda$) = −ive

$\Rightarrow 2\lambda^2 - 2(2\lambda + 1)\lambda + \lambda(\lambda + 1)$

or $-\lambda^2 - \lambda < 0$ or $\lambda(\lambda + 1) > 0$

or $\lambda < -1$ or $\lambda > 0$

**Sol.5 (B)**

The given equation being cubic has three distinct (odd) number of roots between 0 and 1 i.e., a and b hence f(a) and f(b) must be of opposite signs

$\therefore$ f(a). f(b) = f(0). f(1) < 0

or a(a − 2) < 0 or 0 < a < 2

**Sol.6 (A)**

Because of $\sqrt{x}$, $x \geq 0$ ....(1)

$x^2 + 2x - 3 \leq 0 \Rightarrow (x+3)(x-1) \leq 0$

$\therefore -3 \leq x \leq 1$ ....(2)

(1) and (2) together $\Rightarrow$ $0 \leq x \leq 1$ i.e. x lies between 0 and 1 so that middle point is $\frac{1}{2}$.

**Sol.7 (C)**

Here coeffecient of $x^2$ is + ve

$\Rightarrow$ f(a) < 0

Now $f(x) = 2x^2 - 2(2a+1)x + a(a+1)$

$\Rightarrow$ $f(a) = 2a^2 - 2(2a+1)a + a(a+1)$

$= -a^2 - a < 0$

$\Rightarrow$ $-a(a+1) < 0$

$a(a+1) > 0$

$\Rightarrow$ $a \in (-\infty, -1) \cup (0, \infty)$

**Sol.8 (C)**

$3x^2 - 2(a - b + b - c + c - d)x + (a-b)^2$

$+ (b-c)^2 + (c-d)^2 = 0$

$\Rightarrow (x-(a-b))^2 + (x-(b-c))^2$

$+ (x-(c-d))^2 = 0$

$\Rightarrow$ x = a − b = b − c = c − d.

$\therefore$ a, b, c, d are in A.P.

**Sol.9 (A)**

$|f(x) + 2 + 4 - x^2| = |f(x)| + |4 - x^2| + 2$

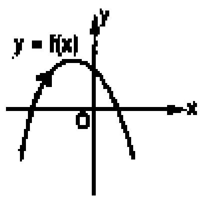
The equation is possible only when each of f(x), $4 - x^2$, 2 are non-positive or all are non-negative. But since 2 is positive, we have

$4 - x^2 \geq 0$ and $f(x) \geq 0$

$\Rightarrow$ $x \in [-2, 2]$

**Sol.10 (C)**

Clearly figure representes a downward parabola having its vertex $\left(-\frac{b}{2a}, -\frac{D}{4a}\right)$ in the second quadrant.




$\Rightarrow$ a < 0, $-\frac{b}{2a} < 0$

$\Rightarrow$ a < 0, −b > 0 or b < 0

$\therefore$ a < 0, b < 0

also, roots are of opposite signs.

$\Rightarrow$ $\frac{c}{a} < 0$ $\Rightarrow$ c > 0.

Hence, a < 0, b < 0, c > 0.