Dear student following is a Moderate level [O ● O] test paper. Score of 9 Marks in 10 Minutes would be a satisfactory performance. Questions 1-6(+3, −1), (All questions have only one option correct)

PASSAGE I :

Let α, β, γ be roots of the equation $ax^3 + bx^2 + cx + d = 0$. To obtain the equation whose roots $f(\alpha), f(\beta), f(\gamma)$ where f is a function, we put $y = f(\alpha)$ and obtain $\alpha = f^{-1}(y)$.

Now, we use the fact that α is root of the equation $ax^3 + bx^2 + cx + d = 0$ to obtain the desired equation.

Note : Desired equation may require some manipulation after substitution.

For example if α, β, γ are roots of $ax^3 + bx^2 + cx + d = 0$ to find equation whose roots are $1/\alpha$, $1/\beta$, $1/\gamma$, we put $\frac{1}{\alpha} = y \Rightarrow \alpha = \frac{1}{y}$. As α is a root of $ax^3 + bx^2 + cx + d = 0$, we get $\frac{a}{y^3} + \frac{b}{y^2} + \frac{c}{y} + d = 0$ or $a + by + cy^2 + dy^3 = 0$.

This is the desired equation.

The same result holds for all polynomial equations.

Q.1 If α, β are roots of $ax^2 + bx + c = 0$, the roots of $a(x - 1)^2 + b(x - 1)(x - 2) + c(x - 2)^2 = 0$ are-

(A) $\frac{2\alpha - 1}{\alpha - 1}, \frac{2\beta - 1}{\beta - 1}$ (B) $\frac{\alpha - 1}{\alpha - 2}, \frac{\beta - 1}{\beta - 2}$

(C) $\frac{\alpha + 1}{\alpha - 2}, \frac{\beta + 1}{\beta - 2}$ (D) $\frac{3\alpha - 1}{\alpha - 1}, \frac{3\beta - 1}{\beta - 1}$

Q.2 If α, β are roots of $px^2 + qx + r = 0$, then the equation whose roots of $\alpha^2 + r/p$ and $\beta^2 + r/p$ is-

(A) $p^3x^2 + pq^2x + r = 0$ (B) $px^2 - qx + r^2 - p = 0$

(C) $p^3x^2 - pq^2x + q^2r = 0$ (D) None of these

Q.3 If α, β, γ are the roots of $x^3 - 13x + 11 = 0$ then equation whose roots are $\beta + \gamma$, $\gamma + \alpha$, $\alpha + \beta$ is-

(A) $x^3 - 13x - 11 + 0$ (B) $x^3 + 13x - 11 = 0$

(C) $x^3 + 13x + 11 = 0$ (D) $x^3 - 13x + 11 = 0$

Q.4 If α, β, γ are roots of $x^3 + 3px + q = 0$, then the equation whose roots are- $(\beta - \gamma)^2$, $(\gamma - \alpha)^2$ and $(\alpha - \beta)^2$ is

(A) $y^3 - 3(p - q)y + pq = 0$

(B) $y^3 + 3(p - q)y - pq = 0$

(C) $y^3 + 3(p + q)y - pq = 0$

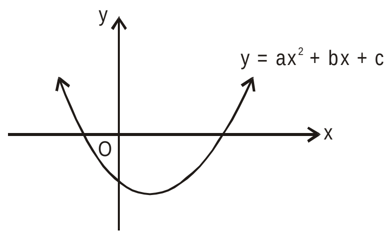
(D) None of these

Q.5 If α, β, γ are roots of $x^3 + 3x + 2 = 0$, then an equation whose roots are $(\alpha - \beta)(\alpha - \gamma)$, $(\beta - \gamma)(\beta - \alpha)$, $(\gamma - \alpha)(\gamma - \beta)$ is-

(A) $y^3 - 6y^2 + 216 = 0$ (B) $y^3 + 9y^2 - 216 = 0$

(C) $y^3 + 3y^2 - 128 = 0$ (D) $y^3 + 6y^2 + 184 = 0$

Q.6 In the given figure, we must have,



(A) $a > 0, c < 0, b < 0$

(B) $a > 0, c > 0, b < 0$

(C) $a > 0, c < 0, b > 0$

(D) $a > 0, c > 0, b > 0$

MATHEMATICS IIT JEE (AUGUST 2nd WEEK CLASS TEST 1) (QUADRATIC EQUATION) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	⬭	⬭	⬭	⬭	3	⬭	⬭	⬭	⬭	5	⬭	⬭	⬭	⬭
2	⬭	⬭	⬭	⬭	4	⬭	⬭	⬭	⬭	6	⬭	⬭	⬭	⬭

ANSWER KEY

Que.	1	2	3	4	5	6
Ans.	A	C	A	D	B	A

SOLUTIONS

Sol.1 (A)

$$a\left(\frac{x-1}{x-2}\right)^2 + b\left(\frac{x-1}{x-2}\right) + c = 0$$

$$\Rightarrow \quad \frac{x-1}{x-2} = \alpha, \beta \Rightarrow x = \frac{2\alpha-1}{\alpha-1}, \frac{2\beta-1}{\beta-1}$$

Sol.2 (C)

Put $y = \alpha^2 + \frac{r}{p}$ or $\alpha = \sqrt{y - \frac{r}{p}}$

$$\therefore p\left(y - \frac{r}{p}\right) + q\sqrt{y - \frac{r}{p}} + r = 0$$

$\Rightarrow p^3y^2 - pq^2y + q^2r = 0$

Sol.3 (A)

As $\alpha + \beta + \gamma = 0$, $\beta + \gamma = -\alpha$.

The desired equation can be obtained by changing x to –x.

Sol.4 (D)

$y = (\beta - \gamma)^2 = (\beta + \gamma)^2 - 4\beta\gamma = \alpha^2 + 4q/\alpha$

$$= 3\left(\frac{q}{\alpha} - p\right)$$

$$\Rightarrow \quad \alpha = \frac{3q}{3p+y}$$

Thus $\frac{27q^3}{(3p+y)^3} + \frac{9pq}{3p+y} + q = 0$

$\Rightarrow \quad 27q^2 + 9p(3p + y)^2 + q(3p + y)^3 = 0.$

Sol.5 (B)

$y = (\alpha - \beta)(\alpha - \gamma)$

$= \alpha^2 - (\alpha\beta + \alpha\gamma) + \beta\gamma$

$= \alpha^2 - (\alpha\beta + \alpha\gamma + \beta\gamma) + 2\beta\gamma$

$y = \alpha^2 - 3 + \frac{2\alpha\beta\gamma}{\alpha}$

$$= \frac{\alpha^3 - 3\alpha - 4}{\alpha} = \frac{-6(\alpha+1)}{\alpha}$$

$$\Rightarrow \quad \alpha = \frac{-6}{6+y}.$$

Thus, $\left(\frac{-6}{6+y}\right)^3 + 3\left(\frac{-6}{6+y}\right) + 2 = 0$

or $\quad y^3 + 9y^2 - 216 = 0$

Sol.6 (A)