Dear student following is a Moderate level [○ ● ○] test paper. Score of 15 Marks in 10 Minutes would be a satisfactory performance. Questions 1-5(+3, –1), 6-7(+6, 0), *(All questions have only one option correct)*

Q.1 In the equation $x^3 + 3Hx + G = 0$, if G and H are real and $G^2 + 4H^3 > 0$, then the roots are-

(A) All real and equal
(B) All real and distinct
(C) One real and two imaginary
(D) All real and two equal

Q.2 The solutions of the quadratic equation $(3|x| - 3)^2 = |x| + 7$ which belongs to the domain of definition of the function $y = \sqrt{x(x-3)}$ are given by-

(A) ± 1/9, ± 2 (B) −1/9, 2
(C) 1/9, −2 (D) −1/9, −2

Q.3 Let $y = \sqrt{\dfrac{(x+1)\,(x-3)}{(x-2)}}$, then all real values of x for which y takes real values, are-

(A) $-1 \le x < 2$ or $x \ge 3$
(B) $-1 \le x < 3$ or $x > 2$
(C) $1 \le x < 2$ or $x \ge 3$
(D) None of these

Q.4 The value of 'c' for which $|\alpha^2 - \beta^2| = \dfrac{7}{4}$, where α and β are the roots of $2x^2 + 7x + c = 0$, is-

(A) 4 (B) 0 (C) 6 (D) 2

Q.5 The roots of the equation $x^4 - 4x^3 + 6x^2 - 4x + 1 = 0$ are -

(A) 1, 1, 1, 1 (B) 2, 2, 2, 2
(C) 3, 1, 3, 1 (D) 1, 2, 1, 2

MATCH THE COLUMN :

Q.6 The number of real roots of

(i) $x^4 + x^2 + 2 = \log_{0.75}(2007)$	(A) 0
(ii) $(14)^{2/x} + 49^{1/x} = (4.25\,(98^{1/x}))$	(B) Infinite
(iii) $\sqrt{x+3-4\sqrt{x-1}} + \sqrt{x-2\sqrt{x-1}} = 1$	(C) 2

Q.7 α, β are roots of $ax^2 + bx + c = 0$. Match equation with its roots

(i) $ax^2 + (2a + b)x + a + b + c = 0$	(A) $\dfrac{\alpha}{\alpha+1}, \dfrac{\beta}{\beta+1}$
(ii) $(a - b + c)x^2 + (b - 2c)x + c = 0$	(B) $\dfrac{\alpha-1}{\alpha+1}, \dfrac{\beta-1}{\beta+1}$
(iii) $(a - b + c)x^2 + 2(a - c)x + a + b + c = 0$	(C) $\alpha - 1, \beta - 1$.

MATHEMATICS IIT JEE (AUGUST 2nd WEEK CLASS TEST 2) (QUADRATIC EQUATION) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D	6	A	B	C	D	7	A	B	C	D
1	○	○	○	○	(i)	○	○	○	○	(i)	○	○	○	○
2	○	○	○	○	(ii)	○	○	○	○	(ii)	○	○	○	○
3	○	○	○	○	(iii)	○	○	○	○	(iii)	○	○	○	○
4	○	○	○	○										
5	○	○	○	○										

ANSWER KEY

Que.	1	2	3	4	5	6			7		
Ans.	C	D	A	C	D	i-A	ii-C	iii-B	i-C	ii-A	iii-B

SOLUTIONS

Sol.1 (C)

Given equation $x^3 + 3Hx + G = 0$ and G and H are real and $G^2 + 4H^3 > 0$.

Let α, β be the roots of given cubic equation.

We know that $\alpha = \left(\frac{-G + \sqrt{G^2 + 4H^3}}{2}\right)^{1/3}$ and

$\beta = \left(\frac{-G - \sqrt{G^2 + 4H^3}}{2}\right)^{1/3}$, since $G^2 + 4H^3 > 0$, therefore the cubic equation has got one real and two imaginary roots.

Sol.2 (D)

Domain of definition of the function

$y = \sqrt{x(x-3)}$ is $x(x - 3) \geq 0$

i.e. $x \leq 0$ or $x \geq 3$ ....(i)

Given equation can be re-written as

$9|x|^2 - 19|x| + 2 = 0$

$\Rightarrow (9|x| - 1)(|x| - 2) = 0$

$\Rightarrow |x| = 2$ or $|x| = 1/9$

$\therefore$ Solution of the given equation are $\pm 2, \pm 1/9$

In the domain (i), the required solutions are $-2, -1/9$.

Sol.3 (A)

$y = \sqrt{\frac{(x+1)(x-3)}{(x-2)}}$

Here x cannot be 2.

$\Rightarrow$ Either both N^r and D^r are positive

$x \geq -1, x \geq 3$ and $x > 2$

$\Rightarrow x \geq 3$ ....(1)

Or N^r is negative and D^r is negative

$x \geq -1$ and $x < 2 \Rightarrow -1 \leq x < 2$ ....(2)

From (1) and (2), $-1 \leq x < 2$ or $x \geq 3$.

Sol.4 (C)

We have $\alpha + \beta = -\frac{7}{2}$ and $\alpha\beta = \frac{c}{2}$

$\therefore |\alpha^2 - \beta^2| = \frac{7}{4} \Rightarrow \alpha^2 - \beta^2 = \pm\frac{7}{4}$

$\Rightarrow (\alpha + \beta)(\alpha - \beta) = \pm\frac{7}{4}$

$\Rightarrow -\frac{7}{2}\sqrt{\frac{49}{4} - 2c} = \pm\frac{7}{4}$

$\Rightarrow \sqrt{49 - 8c} = \mp 1$

$\Rightarrow 49 - 8c = 1 \Rightarrow c = 6$.

Sol.5 (A)

Given equation can be rewritten as

$3x^2 - (a + c + 2b + 2d)x + (ac + 2bd) = 0$

Its discriminant D

$= (a + c + 2b + 2d)^2 - 4.3(ac + 2bd)$

$= \{(a + 2d) + (c + 2b)\}^2 - 12(ac + 2bd)$

$= \{(a + 2d) - (c + 2b)\}^2 + 4(a + 2d)(c + 2b) - 12(ac + 2bd)$

$= \{(a + 2d) - (c + 2b)\}^2 - 8ac + 8ab + 8dc - 8bd$

$= \{(a + 2d) - (c + 2b)\}^2 + 8(c - b)(d - a)$

Which is +ve, since $a < b < c < d$.

Hence roots are real and distinct.

Sol.6 (i-A, ii-C, iii-B)

(i) $x^4 + x^2 + x = (x^2 + 1/2)^2 + 7/4 > 0$
but $\log_{0.75}(2007) < 0$

(ii) Divide both the sides by $98^{1/x}$ to obtain
$t + 1/t = 17/4$
where $t = (1/2)^{1/x}$
$\Rightarrow (1/2)^{1/x} = 4, 1/4 \Rightarrow 2^{1/x} = 2^2, 2^{-2}$
$x = 1/2, -1/2$

(iii) Put $\sqrt{x-1} = t$ to obtain
$|t - 2| + |t - 1| = 1 \Leftrightarrow 1 \leq t \leq 2$

Sol.7 (i-C, ii-A, iii-B)

(i) $a(x^2 + 2x + 1) + b(x + 1) + c = 0$
$\Rightarrow x + 1 = \alpha, \beta \Rightarrow x = \alpha - 1, \beta - 1$

(ii) $ax^2 - bx(x - 1) + c(x - 1)^2 = 0$
$\Rightarrow a\left(-\frac{x}{x-1}\right)^2 + b\left(-\frac{x}{x-1}\right) + c = 0$
$\Rightarrow \frac{-x}{x-1} = \alpha, \beta$
$\Rightarrow x = \frac{\alpha}{\alpha+1}, \frac{\beta}{\beta+1}$

(iii) $a(x + 1)^2 - b(x^2 - 1) + c(x - 1)^2 = 0$
$\Rightarrow a\left(\frac{x+1}{x-1}\right)^2 - b\left(\frac{x+1}{x-1}\right) + c = 0$
$\Rightarrow -\frac{x+1}{x-1} = \alpha, \beta$
$\Rightarrow x = \frac{\alpha-1}{\alpha+1}, \frac{\beta-1}{\beta+1}$