Dear student following is a Moderate level [○ ● ○] test paper. Score of 18 Marks in 15 Minutes would be a satisfactory performance. Questions 1-10(+3, –1), (Questions may have more than one option correct)

Q.1 If the equation $(k - 2)x^2 - (k - 4)x - 2 = 0$ has difference of roots as 3 then the value of k is

(A) 1, 3 (B) 3, 3/2
(C) 2, 3/2 (D) 3/2, 1

Q.2 The roots of equation $2x^2 - 5x + 2 = 0$ are

(A) Negative of each other
(B) Reciprocal to each other
(C) Both roots are zero
(D) None of these

Q.3 If one root of the equation $x^3 + 2x^2 + px + q = 0$ is $(\alpha - 1) - i\beta$ then equation which has one root 2α is

$x^3 - 2x^2 + px - q = 0$.

(A) $x^3 - 2x^2 + px - q = 0$.
(B) $x^3 + 2x^2 + px - q = 0$.
(C) $x^3 - 2x^2 - px - q = 0$.
(D) $x^3 - 2x^2 + px + q = 0$.

Q.4 For real values of x, $2x^2 + 5x - 3 > 0$, if-

(A) $x < -2$ (B) $x > 0$
(C) $x > 1$ (D) None of these

Q.5 If both the roots of the equation $x^2 - 9x + a = 0$ are positive and one is greater than 3 and other is less than 3. Find all positive values of a.

(A) $0 < a < 18$ (B) $-18 < a < 0$
(C) $-18 < a < 18$ (D) $a > 18$

Q.6 If a, b, c are in G.P. then which of the following equations have equal roots—

(A) $(b - c)x^2 + (c - a)x + (a - b) = 0$
(B) $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$
(C) $(a^2 + b^2)x^2 - 2b(a + c)x + (b^2 + c^2) = 0$
(D) None of these

Q.7 If p, q, r be in H.P. and p and r be different having same sign, then the root of the equation $px^2 + 2q + r = 0$ will be

(A) Real (B) Equal
(C) Imaginary (D) None of these

Q.8 If the equation $x^2 + 2(k + 1)x + 9k - 5 = 0$ has only negative roots, then –

(A) $k \le 0$ (B) $k \ge 0$
(C) $k \ge 6$ (D) $k \le 6$

Q.9 If the equation $ax^2 + bx + c = 0$ $(a > 0)$ has two roots α and β such that $\alpha < -2$ and $\beta > 2$, then

(A) $b^2 - 4ac > 0$ (B) $c < 0$
(C) $a + |b| + c < 0$ (D) $4a + 2|b| + c < 0$

Q.10 If the roots of the equation $ax^2 + bx + c = 0$ be $\frac{k+1}{k}$ and $\frac{k+2}{k+1}$, then $(a + b + c)^2 =$

(A) $b^2 - 4ac$. (B) $4ac - b^2$
(C) $c^2 - 4ab$ (D) $a^2 - 4ac$

MATHEMATICS IIT JEE (AUGUST 2nd WEEK CLASS TEST 3) (QUADRATIC EQUATION) ANSWER KEY

Name : .... **Roll No. :** ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	5	○	○	○	○	9	○	○	○	○
2	○	○	○	○	6	○	○	○	○	10	○	○	○	○
3	○	○	○	○	7	○	○	○	○					
4	○	○	○	○	8	○	○	○	○					

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	B	B	A	C	A	C	C	C	ALL	A

SOLUTIONS

Sol.1 (B)

$|(\alpha - \beta)| = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

Now $\alpha + \beta = \frac{(k-4)}{(k-2)}$, $\alpha\beta = \frac{-2}{k-2}$

$\therefore |(\alpha - \beta)| = \sqrt{\left(\frac{k-4}{k-2}\right)^2 + \frac{8}{(k-2)}}$

$= \sqrt{\frac{k^2+16-8k+8(k-2)}{|(k-2)|}}$

$3 = \sqrt{\frac{k^2+16-8k+8k-16}{|(k-2)|}}$

$\Rightarrow 3 = \left|\frac{k}{k-2}\right|$

$\Rightarrow \frac{k}{k-2} = 3$ or $\frac{k}{k-2} = -3$

$k = 3$ or $k = 3/2$

Sol.2 (B)

The roots of the equations $2x^2 - 5x + 2 = 0$ are reciprocal to each other because here $a = c$

Sol.3 (A)

Given equation is $x^3 + 2x^2 + px + q = 0$. ...(1)

One root of the equation is $(\alpha - 1) - i\beta$

$\therefore$ second root will be $(\alpha - 1) + i\beta$.

Let the third root be γ then

Sum of roots $= -\frac{\text{coeff. of } x^2}{\text{coeff. of } x^3}$

$\Rightarrow (\alpha - 1) - i\beta + (\alpha - 1) + i\beta + \gamma = -\frac{2}{1}$

$\Rightarrow \gamma = -2\alpha$

We have to find an equation of which one roots is $-\gamma = 2\alpha$, which can be obtained by substituting x by – x in equation (1).

$-x^3 + 2x^2 - px + q = 0$

$\Rightarrow x^3 - 2x^2 + px - q = 0$

Sol.4 (C)

Discriminant $b^2 - 4ac = 25 + 24 = 49 > 0$

$\Rightarrow$ Roots are real.

$\Rightarrow$ The given expression is positive for those real values of x for which $x \notin (-3, 1/2)$, because $a = 2 > 0$

$\Rightarrow x > 1$ is true.

Sol.5 (A)

Since coeff. of x^2 is positive graph of the given function will be as shown.

Claearly $f(0) > 0$

$a > 0$....(1) **Fig : 56(34)**

& $f(3) < 0$

$9 - 27 + a < 0 \Rightarrow a < 18$ .... (2)

from (1) and (2) $0 < a < 18$

Sol.6 (C)

Here From (A)

$(c - a)^2 4(b - c)(a - b) = 0$

$\Rightarrow (c + a - 2b)^2 = 0$

$\Rightarrow$ a, b, c in A.P.

(B) gives $[b(c - a)]^2 - 4a(b - c)c(a - b) = 0$

$\Rightarrow (ab + bc - 2ac)^2 = 0$

$\Rightarrow$ a, b, c in H.P.

(C) gives $4b^2(a + c)^2 - 4(a^2 + b^2)(b^2 + c^2) = 0$

$\Rightarrow -4(b^2 - ac)^2 = 0$

$\therefore$ a, b, c in G.P.

Note : Take a, b, c as 1, 2, 4 and find the equation by numerical substitution.

Sol.7 (C)

Here p, q, r in H.P. $\Rightarrow q = \frac{2pr}{p+r}$ ...... (1)

Now $D = 4q^2 - 4pr$

$= -4[pr - \left(\frac{2pr}{p+r}\right)^2]$ using (1)

$= -(pr)[2\left(\frac{p-r}{p+r}\right)^2]$

Since $pr > 0$, $p \neq r$ given,

$D \neq 0$ and $D < 0$ Hence the roots are imaginary.

Sol.8 (C)

Let $f(x) = x^2 + 2(k + 1)x + 9k - 5$. Let α, β be the roots of $f(x) = 0$. The equation $f(x) = 0$ will have both negative roots, if –

(i) Disc. ≥ 0

(ii) $\alpha < 0, \beta < 0$, i.e. $(\alpha + \beta) < 0$ and

(iii) $f(0) > 0$

Now, Discriminant ≥ 0

$\Rightarrow 4(k + 1)^2 - 36k + 20 \geq 0$

$\Rightarrow k^2 - 7k + 6 \geq 0$

$\Rightarrow (k - 1)(k - 6) \geq 0$

$\Rightarrow k \leq 1$ or $k \geq 6$ ....(i)

$(\alpha + \beta) < 0 \Rightarrow -2(k + 1) < 0$

$\Rightarrow k + 1 > 0 \Rightarrow k > -1$ .....(ii)

and $f(0) > 0 \Rightarrow 9k - 5 > 0 \Rightarrow k > \frac{5}{9}$ ...(iii)

From (i), (ii) and (iii), we get $k \geq 6$.

Sol.9 (All)

Since the equation has two distinct roots α and β, the discriminant $b^2 - 4ac > 0$

we must have $f(x) = ax^2 + bx + c < 0$ for $\alpha < x < \beta$

Since $\alpha < 0 < \beta$ we must have, $f(0) = c < 0$.

Also as $\alpha < -1, 1 < \beta$ we get, $f(-1) = a - b + c < 0$. and $f(1) = a + b + c < 0$ i.e., $a + |b| + c < 0$.

Next , since $\alpha < -2, 2 < \beta$

$f(-2) = 4a - 2b + c < 0$

and $f(2) = 4a + 2b + c < 0$, i.e. $4a + 2|b| + c < 0$.

Sol.10 (A)

Given equation is $ax^2 + bx + c = 0$ ...(1)

Roots of equation (1) are $\frac{k+1}{k}$ and $\frac{k+2}{k+1}$

$\therefore \frac{k+1}{k} + \frac{k+2}{k+1} = -\frac{b}{a}$ ....(2)

and $\frac{k+2}{k} = \frac{c}{a}$ ....(3)

From (3), $k = \frac{2a}{c-a}$

Putting the value of k in (2), we get

$\frac{c+a}{2a} + \frac{2c}{c+a} = -\frac{b}{a}$ or $\frac{(c-a)^2 + 4ac}{2a(c+a)} = -\frac{b}{a}$

or $a(c + a)^2 + 4a^2c = -2abc - 2a^2b$

or $(c + a)^2 + 4ac = -2bc - 2ab$

or $(c + a)^2 + 2b(c + a) = -4ac$

or $(c + a)(c + a + 2b) = -4ac$

or $(a + b + c - b)(a + b + c + b) = -4ac$

or $(a + b + c)^2 - b^2 = -4ac$

or $(a + b + c)^2 = b^2 - 4ac$