AUGUST 2nd WEEK CLASS TEST 4 QUADRATIC EQUATION MATHEMATICS (FOUNDATION)

Dear student following is a Moderate level [O ● O] test paper. Score of 18 Marks in 15 Minutes would be a satisfactory performance. Questions 1-10(+3, –1), (Questions may have more than one option correct)

Q.1 If $a_1, a_2, a_3, \ldots a_n$, $(n \geq 2)$ are real and $(n-1)\,a_1^2 - 2n\,a_2 < 0$, then at least two roots of the equation $x^n + a_1 x^{n-1} + a_2 x^{n-2} \ldots + a_n = 0$ are

(A) Real (B) Rational
(C) Irrational (D) Imaginary

Q.2 If $a, b, c \in R$, $a \neq 0$ and the quadratic equation $ax^2 + bx + c = 0$ has no real roots, then $(a + b + c)\,c > 0$.

(A) > 0 (B) < 0
(C) ≥ 0 (D) ≤ 0

Q.3 If the roots of $x^2 - ax + b = 0$ are real and differ by a quantity which is less than $c\,(c > 0)$. Then b lies between

(A) $\left(\frac{a^2 - c^2}{4}\right)$ & $\frac{a^2}{4}$ (B) $\left(\frac{c^2 - a^2}{4}\right)$ & $\frac{a^2}{4}$
(C) $\left(\frac{a^2 - c^2}{4}\right)$ & $\frac{c^2}{4}$ (D) None of these

Q.4 If $c \neq 0$ and the equation $\frac{p}{2x} = \frac{a}{x+c} + \frac{b}{x-c}$ has two equal roots, then p can be

(A) $\left(\sqrt{a} - \sqrt{b}\right)^2$ (B) $\left(\sqrt{a} + \sqrt{b}\right)^2$
(C) $a + b$ (D) $a - b$

Q.5 $\frac{8x^2 + 16x - 51}{(2x-3)(x+4)} > 3$ if x is such that –

(A) $x < -4$ (B) $-3 < x < 3/2$
(C) $x > 5/2$ (D) All three correct

Q.6 The difference of maximum and minimum value of $\frac{x^2 + 4x + 9}{x^2 + 9}$ =

(A) 4/3 (B) 3/4
(C) –4/3 (D) 5/3

Q.7 If r be the ratio of the roots of the equation $ax^2 + bx + c = 0$, then $\frac{(r+1)^2}{r}$ =

(A) $\frac{b^2}{ac}$ (B) $\frac{b}{ac}$
(C) $\frac{ac}{b^2}$ (D) $\frac{c^2}{ab}$

Q.8 The number of points (x, y), (where x and y both are perfect squares of integers) on the parabola $y^2 = px$, p being a prime number, is:

(A) One (B) Zero
(C) Two (D) Infinite

Q.9 $\sin x + \cos x = y^2 - y + a$ has no value of x for any y, if 'a' belongs to

(A) $(0, \sqrt{3})$ (B) $(-\sqrt{3}, 0)$
(C) $(-\infty, -\sqrt{3})$ (D) $(\sqrt{3}, \infty)$

Q.10 If a, b, c be the sides of ΔABC and equations $ax^2 + bx + c = 0$ and $5x^2 + 12x + 13 = 0$ have a common root, then $\angle C$ is

(A) 60^0 (B) 90^0
(C) 120^0 (D) 45^0

MATHEMATICS IIT JEE (AUGUST 2nd WEEK CLASS TEST 4) (QUADRATIC EQUATION) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	5	○	○	○	○	9	○	○	○	○
2	○	○	○	○	6	○	○	○	○	10	○	○	○	○
3	○	○	○	○	7	○	○	○	○					
4	○	○	○	○	8	○	○	○	○					

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	D	A	A	A,B	D	A	A	A	D	B

SOLUTIONS

Sol.1 (D)

Let $\alpha_1, \alpha_2, \ldots.\alpha_n$, are the roots of the given equation then $\Sigma\alpha_1 = \alpha_1 + \alpha_2 \ldots.\alpha_n = -a_1$
and $\Sigma\alpha_1\alpha_2 = \alpha_1\alpha_2 + \alpha_2\alpha_3 + \ldots. + \alpha_{n-1}\alpha_n = a_2$

$\therefore (n-1)\ a_1^2 - 2na_2$

$= (n-1)(\Sigma\alpha_1)^2 - 2n\ \Sigma\alpha_1\alpha_2$

$= \sum\limits_{1\le i<j\le n}\sum (\alpha_i - \alpha_j)^2$

But given that $(n-1)\ a_1^2 - 2na_2 < 0$

$\therefore \sum\limits_{1\le i<j\le n}\sum (\alpha_i - \alpha_j)^2 < 0$

Which is true only when atleast two roots are imaginary.

Sol.2 (A)

Let $f(x) = ax^2 + bx + c$

Since equation $ax^2 + bx + c = 0$ i.e. equation $f(x) = 0$ has no real root, therefore $f(x)$ will have same sign for all real values of x.

$\Rightarrow$ f(0) and f(1) will have same sign.

$\Rightarrow f(1).\ f(0) > 0 \quad \Rightarrow (a + b + c) > 0$

Sol.3 (A)

As root of $x^2 - ax + b = 0$ are Real and different

$a^2 - 4b > 0$

$b < \frac{a^2}{4}$ ..... (i)

Let the roots are a and b

given $\alpha - \beta < c$

$\sqrt{(\alpha+\beta)^2 - 4\alpha\beta < c}$

$\sqrt{a^2 - 4b} < c$

$a^2 - 4b < c^2 \quad (c > 0,\ a^2 - 4b > 0)$

$b > \frac{1}{4}(a^2 - c^2)$ .....(ii)

from (i) and (ii)

$\frac{1}{4}(a^2 - c^2) < b < \frac{a^2}{4}$

Sol.4 (A,B)

We can write the given equation as

$\frac{p}{2x} = \frac{(a+b)x + c(b-a)}{x^2 - c^2}$

or $\quad p(x^2 - c^2) = 2(a + b)\ x^2 - 2c\ (a - b)\ x$

or $\quad (2a + 2b - p)\ x^2 - 2c\ (a - b)\ x + pc^2 = 0$

For this equation to have equal roots

$c^2 (a - b)^2 - pc^2 (2a + 2b - p) = 0$

$\Rightarrow (a - b)^2 - 2p\ (a + b) + p^2 = 0$

$\Rightarrow [p - (a + b)]^2 = (a + b)^2 - (a - b)^2 = 4ab$

$\Rightarrow p - (a + b) = \pm 2\sqrt{ab}$

$\Rightarrow p = a + b \pm 2\sqrt{ab} = \left(\sqrt{a} \pm \sqrt{b}\right)^2$

Sol.5 (D)

Consider $\frac{8x^2 + 16x - 51}{(2x-3)(x+4)} - 3 > 0$

$\Rightarrow \frac{2x^2 + x - 15}{2x^2 + 5x - 12} > 0$

$\Rightarrow \frac{(2x-5)(x+3)}{(2x-3)(x+4)} > 0$

Hence both Nr and Dr are positive if $x < -4$ or $x > \frac{5}{2}$ and both negative if $-3 < x < \frac{3}{2}$

Sol.6 (A)

Let $y = \frac{x^2 + 4x + 9}{x^2 + 9}$

$(y - 1)\ x^2 - 4x + 9\ (y - 1) = 0$

For real value of x, $D \ge 0$

$16 - 36\ (y - 1)^2 \ge 0$

$4 - 9\ (y - 1)^2 \ge 0$

$\{2 - 3\ (y - 1\}\{2 + 3\ (y - 1)\} \ge 0$

$(5 - 3y)(3y - 1) \ge 0$

$\frac{1}{3} \le y \le \frac{5}{3}$

$\therefore$ difference of maximum and minimum value is $\frac{5}{3} - \frac{1}{3} = \frac{4}{3}$

Sol.7 (A)

Given equation is $ax^2 + bx + c = 0$ ...(1)

Let the roots of equation (1) be α and $r\alpha$, then

$\alpha + r\alpha = -\frac{b}{a}$ .....(2)

$r\alpha^2 = \frac{c}{a}$ ....(3)

From (2), $\alpha = -\frac{b}{a(r+1)}$ ....(4)

putting the value of α in (3), we get

$\frac{rb^2}{a^2(r+1)^2} = \frac{c}{a}$ or, $\frac{b^2}{ac} = \frac{(r+1)^2}{r}$

Sol.8 (A)

If 'x' is a perfect square, then 'px' will be a perfect square only if 'p' is a perfect square which is not possible as 'p' is a prime number. Hence y cannot by a perfect suqre.

So number of such points will be only one (0, 0).

Sol.9 (D)

Since, $y^2 - y + a = \left(y - \frac{1}{2}\right)^2 + a - \frac{1}{4}$

and $-\sqrt{2} \le \sin x + \cos x \le \sqrt{2}$, given equation will have no real value of x for any y, if $a - \frac{1}{4} > \sqrt{2}$

i.e., $a \in \left(\sqrt{2} + \frac{1}{4}, \infty\right) \Rightarrow a \in (\sqrt{3}, \infty)$

$\left\{\text{as } \sqrt{2} + \frac{1}{4} < \sqrt{3}\right\}$.

Sol.10 (B)

Since, $5x^2 + 12x + 13 = 0$ has imaginary roots as,

$D = 144 - 4 \times 5 \times 13 = -116 < 0$.

So, both roots of $ax^2 + bx + c = 0$ and

$5x^2 = 12x + 13 = 0$ are common.

$\therefore \quad \frac{a}{5} = \frac{b}{12} = \frac{c}{13} \Rightarrow a^2 + b^2 = c^2$

$\angle C = 90^0$.