Dear student following is an Easy level [●○○] test paper. Score of 24 Marks in 15 Minutes would be a satisfactory performance. Questions 1-10(+3,–1). (All questions have only one option correct).

Q.1 Condition that the roots of the equation $x^3 - px^2 + qx - r = 0$ are in H.P.

(A) $27r^2 - 9prq + 2q^3 = 0$
(B) $27p^2 - 9rpq + 2q^3 = 0$
(C) $27r^2 - 9prq + 2p^3 = 0$
(D) $27p^2 - 9rpq + 2r^3 = 0$

Q.2 If α, β are roots of the equation $2x^2 + 6x + b = 0$ $(b < 0)$ then $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ is less than-

(A) 2 (B) – 2
(C) 18 (D) None of these

Q.3 If the roots of the quadratic equation $x^2 - 4x - \log_3 a = 0$ are real, then the least value of a is-

(A) 81 (B) $\frac{1}{81}$
(C) $\frac{1}{64}$ (D) None of these

Q.4 If equation $(3x)^2 + (27 \times 3^{1/p} - 15)x + 4 = 0$ has equal roots then p =

(A) 0 (B) 2
(C) $-\frac{1}{2}$ (D) None of these

Q.5 If the roots of the equation $ax^2 + bx + c = 0$ are real and distinct, then-

(A) Both roots are greater than $-\frac{b}{2a}$
(B) Both roots are less then $-\frac{b}{2a}$
(C) One of the roots exceeds $-\frac{b}{2a}$
(D) None of these

Q.6 The values of the parameter a for which the quadratic equations $(1 - 2a)x^2 - 6ax - 1 = 0$ and $ax^2 - x + 1 = 0$ have at least one root in common are-

(A) $0, \frac{1}{2}$ (B) $\frac{1}{2}, \frac{2}{9}$
(C) $\frac{2}{9}$ (D) $0, \frac{1}{2}, \frac{2}{9}$

Q.7 Let x_1, x_2 be the roots of the equation $x^2 - 3x + p = 0$ and x_3, x_4 be the roots of the equation $x^2 - 12x + q = 0$. If the number x_1, x_2, x_3, x_4 (in order) form an increasing G.P., then-

(A) p = 2, q = 16 (B) p = 2, q = 32
(C) p = 4, q = 16 (D) p = 4, q = 32

Q.8 The condition that one root of the equation $ax^2 + bx + c = 0$ may be double of the other, is-

(A) $b^2 = 9ac$ (B) $2b^2 = 9ac$
(C) $2b^2 = ac$ (D) $b^2 = ac$

Q.9 If the roots of the equation $x^2 - px + q = 0$ differ by unity, then-

(A) $p^2 = 4q$ (B) $p^2 = 4q + 1$
(C) $p^2 = 4q - 1$ (D) None of these

Q.10 The number of real roots of the equation $|x|^2 - 3|x| + 2 = 0$ is-

(A) 4 (B) 3 (C) 2 (D) 1

MATHEMATICS IIT JEE (JULY 3rd WEEK CLASS TEST 4) (QUADRATIC EQUATION) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○
										10	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	A	B	B	C	C	C	B	B	B	A

SOLUTIONS

Sol.1 (A)

The equation whose roots are reciprocal of the given equation is

$$\frac{1}{x^3} - \frac{p}{x^2} + \frac{q}{x} - r = 0$$

$\Rightarrow$ $rx^3 - qx^2 + px - 1 = 0$ ...... (1)

Since roots of given equation are in H.P., this implies roots of equation (1) are in A.P.

Let the roots of equation (1) be

$a - d, a, a + d$

$\therefore$ $(a - d) + a + (a + d) = -\left(-\frac{q}{r}\right)$

$\Rightarrow$ $3a = \frac{q}{r}$ $\Rightarrow$ $a = \frac{q}{3r}$

Now since a is root of eq. (1), so

$ra^3 - qa^2 + pa - 1 = 0$

$\Rightarrow$ $r\left(\frac{q}{3r}\right)^3 - q\left(\frac{q}{3r}\right)^2 + p\left(\frac{q}{3r}\right) - 1 = 0$

$\Rightarrow$ $27r^2 - 9pqr + 2q^3 = 0$

Sol.2 (B)

Given α, β are roots of $2x^2 + 6x + b = 0$

$\Rightarrow$ $\alpha + \beta = -\frac{6}{2} = -3$ and $\alpha\beta = \frac{b}{2}$

Since $b < 0$, discriminant $D = 36 - 4b > 0$

$\Rightarrow$ α, β are real

$\therefore$ $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$

$= \frac{9 - b}{b/2} = \frac{18}{b} - 2$

$\Rightarrow$ $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} < -2$ $\because$ $b < 0$

Sol.3 (B)

Given roots of the quadratic equation $x^2 - 4x - \log_3 a = 0$ are real

$\Rightarrow$ Disc. $D \geq 0$

$\Rightarrow$ $16 + 4\log_3 a \geq 0$

$\Rightarrow$ $\log_3 a \geq -4$

$\Rightarrow$ $a \geq 3^{-4}$

$\Rightarrow$ $a \geq \frac{1}{81}$

$\Rightarrow$ Least value of a is $\frac{1}{81}$.

Sol.4 (C)

Condition for equal roots is D = 0

$\Rightarrow$ $(27 \times 3^{1/p} - 15)^2 - 4 \times 4 \times 9 = 0$

$\Rightarrow$ $27 \times 3^{1/p} - 15 = (144)^{1/2}$

$\Rightarrow$ $27 \times 3^{1/p} - 15 = \pm 12$

$\Rightarrow$ $27 \times 3^{1/p} = 27$ or 3

$\Rightarrow$ $3^{1/p} = 1$ or $1/9$

$\Rightarrow$ $1/p = 0$ or -2

Thus $\frac{1}{p}$ cannot be zero.

$\Rightarrow$ $p = -\frac{1}{2}$

Sol.5 (C)

The roots of the given equation are

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a},$$

$$\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Given α, β are real and distinct

$\therefore$ $b^2 - 4ac > 0$

Now, if $a > 0$, then $\beta > -\frac{b}{2a}$

and if $a < 0$, then $\alpha > -\frac{b}{2a}$

Hence, one of the roots exceeds $-\frac{b}{2a}$

Sol.6 (C)

$\because$ For $a = 0$ or $a = \frac{1}{2}$, one of the quadratic equations become linear

$\Rightarrow$ $a \neq 0$ or $\frac{1}{2}$

Hence, the only answer is $a = \frac{2}{9}$

Sol.7 (B)

$\because$ x_1, x_2 are roots of $x^2 - 3x + p = 0$

$\Rightarrow$ $x_1 + x_2 = 3$ and $x_1 x_2 = p$

and x_3, x_4 are roots of $x^2 - 12x + q = 0$

$\Rightarrow$ $x_3 + x_4 = 12$ and $x_3 x_4 = q$

given x_1, x_2, x_3, x_4 are in increasing G.P.

let $x_1 = a, x_2 = ar, x_3 = ar^2, x_4 = ar^3$

Thus $a + ar = 3$ and $ar^2 + ar^3 = 12$

$\Rightarrow$ $a(1 + r) = 3$ and $ar^2(1 + r) = 12$

$\Rightarrow$ $r = \pm 2$

$\therefore$ $r = 2$ $\because$ $r \neq -2$

so $a = 1$

$\therefore$ $x_1 = 1, x_2 = 2, x_3 = 4, x_4 = 8$

Hence $p = 2$ and $q = 32$

Sol.8 (B)

Let the roots be α and 2α. Then

$$3\alpha = \frac{-b}{a},\ 2\alpha^2 = \frac{c}{a} \Rightarrow 2b^2 = 9ac.$$

Sol.9 (B)

Let the roots be α and $\alpha + 1$. Then,

$$\alpha + \alpha + 1 = p \Rightarrow \alpha = \frac{p-1}{2} \quad ...(1)$$

and $\alpha(\alpha + 1) = q \Rightarrow \alpha^2 + \alpha = q$ ...(2)

Eliminating α, from (1) and (2),

$$\left(\frac{p-1}{2}\right)^2 + \left(\frac{p-1}{2}\right) = q$$

$\Rightarrow$ $p^2 - 2p + 1 + 2p - 2 = 4q$

$\Rightarrow$ $p^2 = 4q + 1.$

Sol.10 (A)

We have : $|x|^2 - 3|x| + 2 = 0$

$\Rightarrow$ $(|x| - 1)(|x| - 2) = 0$

$\Rightarrow$ $|x| = 1, 2, \Rightarrow x = \pm 1, \pm 2$