Dear student following is a Moderate level [O ● O] test paper. Score of 12 Marks in 10 Minutes would be a satisfactory performance. Questions 1-5(+3, –1), 6(+6, 0) (All questions have only one option correct)

Q.1 The roots of the equation $x^2 + ax + b = 0$ are both real and greater than the real cube root of unity. If $t = a + b + 1$ then t satisfies the condition-

(A) $t < 0$ (B) $t > 0$
(C) $t = 0$ (D) None of these

Q.2 Let us consider the quadratic equation $ax^2 + bx + c = 0$ in which we define $D = b^2 - 4ac$ and decide the roots as

$$D = \begin{cases} \text{(i)} > 0, & \text{real and distinct} \\ \text{(ii)} = 0, & \text{real and equal} \end{cases}$$

$$\begin{cases} \text{(iii)} < 0, & \text{complex root exists as a conjugate pair} \\ \text{(iv)} & \text{Not a perfect square, roots irrational conjugates} \end{cases}$$

Let tangents at P and Q on the parabola $y^2 = 4x$ meet in T. Let S be the focus let SP, ST, SQ are equal to ℓ, m, n respectively then roots of the equation $\ell x^2 + 2mx + n = 0$ are-

(A) Irrational pair (B) Complex pair
(C) Real and distinct (D) None of these

Q.3 If $2x^2 + 3x + 4 = 0$ and $ax^2 + bx + c = 0$ have a common root and a, b, c $\in$ N then the minimum value of a + b + c equals-

(A) 8 (B) 9 (C) 10 (D) 12

Q.4 The quadratic equation $x^2 - 2x - \lambda = 0$, $\lambda \neq 0$

(A) Cannot have a real root if $\lambda < -1$
(B) Can have a rational root if λ is a perfect square
(C) Cannot have an integral root if $n^2 - 1 < \lambda < n^2 + 2n$ where n = 0, 1, 2, 3,.....
(D) Both (A) and (C)

Q.5 The roots of the quadratic equation $8x^2 - 10x + 3 = 0$ are α and β^2, where $\beta^2 > \frac{1}{2}$ then the equation whose roots are $(\alpha + i\beta)^{100}$ and $(\alpha - i\beta)^{100}$ is-

(A) $x^2 - x + 1 = 0$ (B) $x^2 + x + 1 = 0$
(C) $x^2 - x - 1 = 0$ (D) $x^2 + x - 1 = 0$

Q.6 Match the entries of List A and List B

List A	List B
(i) If one root of the equation $ax^2 + bx + c = 0$ be square of the other, then	(A) $bx^2 - 2a\sqrt{a}\,x + a^2 = 0$
(ii) If α, β be the roots of the equation $x^2 + x + 1 = 0$, then the equation whose roots are α^{19}, β^7 is	(B) $\frac{3b}{2a}$
(iii) A quadratic equation whose roots are $\frac{a}{\sqrt{a} \pm \sqrt{a-b}}$ is	(C) $b^3 + ac(a + c) = 3abc$

MATHEMATICS IIT JEE (AUGUST 3rd WEEK CLASS TEST 1) (QUADRATIC EQUATION) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D	6.	A	B	C
1	○	○	○	○	4	○	○	○	○	(i)	○	○	○
2	○	○	○	○	5	○	○	○	○	(ii)	○	○	○
3	○	○	○	○						(iii)	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6		
Ans.	B	D	B	D	B	i-C	ii-B	iii-A

SOLUTIONS

Sol.1 (B)

Real root of cube root of unity is 1 i.e. both roots are > 1 as t = a + b + 1 and a + b + 1 = 0 for x = 1 but it is given that

x > 1 which mean a + b + 1 > 0

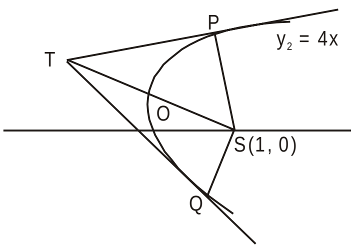
$\Rightarrow$ t = a + b + 1 > 0

$\Rightarrow$ t > 0

Sol.2 (D)

If $P(t_1)$ and $Q(t_2)$ then

$T = (t_1t_2,\ t_1 + t_2)$ and



S = (1, 0) for parabola $y^2 = 4x$

$\ell = SP = 1 + t_1^2$, $n = SQ = 1 + t_2^2$

$\therefore\ m^2 = ST = (t_1t_2 - 1)^2 - (t_1 + t_2)^2$

$= (1 + t_1^2)(1 + t_2^2)$

$m^2 = \ell n \Rightarrow 4m^2 - 4\ell n = 0$

$\therefore\ D = 4m^2 - 4\ell n = 0$

$\Rightarrow$ roots of equation $\ell x^2 + 2mx + n = 0$ having real and equal roots.

Sol.3 (B)

The equation $2x^2 + 3x + 4 = 0$ has its discriminant given by D = 9 – 4 × 4 × 2 = 9 – 32 < 0.

Hence roots are non-real. Since complex roots occur in pairs, so both the roots of the equation $ax^2 + bx + c$ are same as those of $2x^2 + 3x + 4 = 0$.

Thus coefficients of $ax^2 + bx + c$ are proportional to $2x^2 + 3x + 4 = 0$.

i.e. $\frac{a}{2} = \frac{b}{3} = \frac{c}{4} = \lambda$ (say) where $\lambda > 0$

$a + b + c = 9\lambda$

As a, b, c are natural, minimum value of

a + b + c = 9.

Sol.4 (D)

$D = 4 + 4\lambda$, $D < 0 \Rightarrow 4 + 4\lambda < 0 \Rightarrow \lambda < -1$.

$D = 4(1 + \lambda) = 4(1 + p^2) \neq$ perfect square.

If α is an integral root, $\alpha^2 - 2\alpha - \lambda = 0$

or $\lambda = \alpha^2 - 2\alpha = (\alpha - 1)^2 - 1$

$\therefore\ \lambda = (\text{integer})^2 - 1$.

But if $n^2 - 1 < \lambda < n^2 + 2n$ then

$n^2 - 1 < \lambda < (n + 1)^2 - 1$

$\therefore\ \lambda \neq (\text{integer})^2 - 1$.

Sol.5 (B)

$f(x) = (2x - 1)(4x - 3)\ \therefore\ x = \frac{1}{2}, \frac{3}{4}$

$\therefore\ \alpha = \frac{1}{2}$ and $\beta^2 = \frac{3}{4}\left(\beta^2 > \frac{1}{2} = \frac{2}{4}\right)$

$\alpha + i\beta = \frac{1}{2} + \frac{i\sqrt{3}}{2} = re^{i\theta} = e^{i\pi/3}$

$\because\ r = 1,\ \theta = \frac{\pi}{3}$

$\therefore\ \alpha - i\beta = e^{i\pi/3}$ (Conjugate)

$(\alpha + i\beta)^{100} = e^{i100\pi/3}$

$= e^{i.33\pi} \cdot e^{i\pi/3} = -e^{i\pi/3}$

as $e^{i33\pi} = e^{i.32\pi} \cdot e^{i\pi} = 1(-1) = -1$

$(\alpha - i\beta)^{100} = -e^{-i\pi/3}$ (Conjugate)

$\text{Sum} = -\left(e^{i\pi/3} + e^{-i\pi/3}\right)$

$= -2\cos\frac{\pi}{3} = -1$

Product = 1

$\therefore$ Required equation is $x^2 + x + 1 = 0$.

Sol.6

(i) (C)

$S = \alpha + \alpha^2 = -\frac{b}{a}$, $P = \alpha^3 = \frac{c}{a}$

$\therefore \quad (\alpha + \alpha^2)^3 = -\frac{b^3}{a^3}$

or $\quad \alpha^3 + \alpha^6 + 3\alpha.\alpha^2(\alpha + \alpha^2) = -\frac{b^3}{a^3}$

or $\quad \frac{c}{a} + \frac{c^2}{a^2} + \frac{3c}{a}\left(-\frac{b}{a}\right) = -\frac{b^3}{a^3}$

or $\quad b^3 + ac(a + c) = 3abc$

(ii) (B)

Roots of $x^2 + x + 1 = 0$ are ω, ω^2

$\alpha = \omega$, $\beta = \omega^2$,

$\therefore \quad \alpha^{19} = \omega^{19} = \omega$

$\beta^7 = \omega^{14} = \omega^2$

(iii) (A)

On rationalizing, the roots are

$\frac{a}{b}\left[\sqrt{a} \pm \sqrt{(a-b)}\right]$

$\therefore \quad S = \frac{2a\sqrt{a}}{b}$, $P = \frac{a^2}{b}$