Dear student following is a tough level [○ ○ ●] test paper. Score of 15 Marks in 15 Minutes would be a satisfactory performance. Questions 1-10(+3, –1) (All questions have only one option correct)

Q.1 There are n lines drawn in a plane such that no two of them are parallel and no three of them are concurrent. The number of different points at which these lines will cut is

(A) $\sum_{k=1}^{n-1} k$ (B) $n(n-1)$

(C) n^2 (D) ${}^{n-1}C_2$

Q.2 Number of 6-digit telephone numbers, which can be constructed with digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, if each number starts with 35 and no digit appears more than once is

(A) 1680 (B) 8! (C) 6! (D) 6.6!

Q.3 In a plane there are 37 straight lines of which 13 pass through the point A and 11 pass through the point B. Besides, no three lines pass through one point, no line passes through both points A and B, and no two are parallel. Then the number of intersection points the lines have is equal to

(A) 535 (B) 601

(C) 728 (D) None of these

Q.4 From six gentlemen and four ladies, a commitete of five is to be formed. Number of ways in which this can be done if the committee is to include at least one lady is

(A) 252 (B) 246 (C) 248 (D) 250

Q.5 Four students of class X, five students of class XI and six students of class XII sit in row. The number of ways, they can sit in a row so that students belonging to the same class are together is :

(A) 3! 4! 5! 6! (B) 3 × 4! 5! 6!

(C) 4! 5! 6! (D) $\frac{15!}{4!5!6!}$

Q.6 The number of rectangles exculding squares from a rectangle of size 9 × 6 is

(A) 391 (B) 791

(C) 842 (D) None of these

Q.7 A teaparty is arranged of 16 people along two sides of a large table with 8 chairs on each side. Four men want to sit on one particular side and two on the other side. The number of ways in which they can be seated is

(A) $\frac{6!8!10!}{4!6!}$ (B) $\frac{8!8!10!}{4!6!}$

(C) $\frac{8!8!6!}{6!4!}$ (D) None of these

Q.8 A five digit number divisible by 3 is to be formed using the numerals 0, 1 , 2, 3, 4 and 5 without repetition. The total number of ways this can be done is

(A) 216 (B) 240 (C) 600 (D) 3125

Q.9 Given that n is odd, the number of ways in which three numbers in A.P. can be selected from, 1, 2, 3,....n is

(A) $\frac{(n-1)^2}{2}$ (B) $\frac{(n+1)^2}{4}$

(C) $\frac{(n+1)^2}{2}$ (D) $\frac{(n-1)^2}{4}$

Q.10 The number of ways in which 52 cards can be divided into 4 sets, three of them having 17 cards each and the fourth one having just one card

(A) $\frac{52!}{(17!)^3}$ (B) $\frac{52!}{(17!)^3 3!}$

(C) $\frac{51!}{(17!)^3}$ (D) $\frac{51!}{(17!)^3 3!}$

MATHEMATICS IIT JEE (SEPT.3rd WEEK CLASS TEST 3) (PERMUTATION & COMBINATION) ANSWER KEY

Name : .... **Roll No. :** ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○
										10	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	A	A	A	B	A	B	B	A	D	B

SOLUTIONS

Sol.1 (A)

The number of points intersection of n non parallel lines is ${}^nC_2 = \frac{n(n-1)}{2}$

And $\sum_{k=1}^{n-1} k = 1 + 2 + 3 + ... + (n - 1)$

$= \frac{n-1}{2}(n - 1 + 1) = \frac{n(n-1)}{2}$

Sol.2 (A)

Since each number consisting of 6 digits starts with 35, 3 and 5 are fixed in the first and second places.

The other four places can be filled up with remaining 8 digits in ${}^8P_4$ ways = 8.7.6.5 = 1680 ways. Hence the required numbers of telephone number is 1680.

Sol.3 (A)

The number of points of intersection of 37 straight lines is ${}^{37}C_2$. But 13 of them pass through the point A. Therfore instead of getting ${}^{13}C_2$ points we get merely one point. Similarly 11 straight lines out of the given 37 straight lines intersect at B. Therfore instead of getting ${}^{11}C_2$ Points, we get only one point. Hence, the number of intersection points of the lines is

${}^{37}C_2 - {}^{13}C_2 - {}^{11}C_2 + 2 = 535.$

Sol.4 (B)

We can select 5 members for the committee to include at least one lady in the following four ways :

(1) 1 lady and 4 gentleman

(2) 2 ladies and 3 gentlemen

(3) 3 ladies and 2 gentlemen

(4) 4 ladies and 1 gentleman.

Hence the number of committees

$= {}^4C_1 \times {}^6C_4 + {}^4C_2 \times {}^6C_3 + {}^4C_3 \times {}^6C_2 + {}^4C_4 \times {}^6C_1$

$= 60 + 120 + 60 + 6 = 246$

Sol.5 (A)

Treating them as three groups, the three groups can be arranged in 3 ways. Thereafter each group can be arranged internally. Total number = 3! × 4! × 5! × 6!

Sol.6 (B)

Here n = 6 and p = 9

∴ No. of rectangles excluding square

$\frac{6.9}{4}(6 + 1)(9 + 1) - \sum_{r=1}^{6}(7 - r)(10 - r)$

$= 945 - \sum_{r=1}^{6}(70 - 17r + r^2)$

$= 945 - 154 = 791$

Sol.7 (B)

There are 8 chairs on each side of the table. Let the sides be represented by A and B. Let four persons sit on side A, then number of ways of arranging 4 persons on 8 chairs on side A = ${}^8P_4$ and then two person sit on side B. The number of ways of arranging 2 persons on 8 chairs on side B = ${}^8P_2$ and the remaining 10 persons can be arranged in remaining 10 chairs in 10! ways.

Hence the total numebr of ways in which the persons can be arranged

$= {}^8P_4 \times {}^8P_2 \times 10! = \frac{8!8!10!}{4!6!}$

Sol.8 (A)

If a number is divisible by 3, the sum of the digits in it must be multiple of 3. The sum of the given six numerals is 0 + 1 + 2 + 3 + 4 + 5 = 15. So to make a five digit number divisible by 3 we can either exclude 0 or 3. If 0 is left out, then 5! = 120 number of ways are possible. If 3 is left out, then the number of ways of making a five digit numbers is 4 × 4! = 96, because 0 cannot be placed in the first place from left, as it will give a number of four digits. Hence, total number = 120 + 96 = 216.

Sol.9 (D)

There are in the set (1, 2, 3,....n) (n being odd), $\frac{n-1}{2}$ even numbers, $\frac{n+1}{2}$ odd numbers and for an A.P., the sum of the extremes is always even and hence the choice is either both even or both odd and this may be done in

$$^{\frac{n-1}{2}}C_2 + {}^{\frac{n+1}{2}}C_2 = \frac{(n-1)^2}{4} \text{ ways}$$

Note that, if a, b, c are in A.P. a + c = 2b. Hence, if a, b, c are integer the sum of extreme digits (a and c) is even.

Sol.10 (B)

Here we have to divide 52 cards into 4 sets, three of them having 17 cards each the fourth one having just one card. First we divide 52 cards into two groups of 1 card and 51 cards, this can be done in $\frac{52!}{1!51!}$ ways.

Now every group of 51 cards can be divided into 3 groups of 17 each in $\frac{51!}{(17!)^3 3!}$

Hence ther required number of ways

$$= \frac{52!}{1!51!} \cdot \frac{51!}{(17!)^3 3!} = \frac{52!}{(17!)^3 3!}$$