Dear student following is an Easy level [●○○] test paper. Score of 24 Marks in 15 Minutes would be a satisfactory performance. Questions 1-10(+3,–1). (All questions have only one option correct).

Q.1 All the three roots of equation $x^3 - 3x + 1 = 0$ lie on the interval

(A) $[-2, 0]$ (B) $[-1, 1]$
(C) $[-2, 2]$ (D) $[-1, 2]$

Q.2 The roots of equation $ax^2 + 2bx + c = 0$ are equal. Then $ax^2 + 2bx + c + k$ is positive for every real value of x if

(A) $k > 0$ (B) $a > 0, k > 0$
(C) $k < 0$ (D) $c > 0, k > 0$

Q.3 Let $\alpha/(\alpha - 1)$ and $\beta/(\beta - 1)$ be the roots of $x^2 + ax + b = 0$. Then $1/\alpha$ and $1/\beta$ are the roots of

(A) $bx^2 + ax + 1 = 0$ (B) $bx^2 - ax + 1 = 0$
(C) $bx^2 + (a + 2b)x + a + b + 1 = 0$
(D) $bx^2 - (a + 2b)x + a + b + 1 = 0$

Q.4 If the sum of roots of the equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals then a/c, b/a, c/b are in

(A) A. P. (B) G. P.
(C) H. P. (D) None of these

Q.5 Let α and β be the roots of $x^2 + (2b - a^2)x + b^2 = 0$, and α and β be those of $x^2 + (2a - b^2)x + a^2 = 0$. If $\sqrt{\alpha} - \sqrt{\beta} = \sqrt{\alpha'} - \sqrt{\beta'}$, then a + b is equal to

(A) –4 (B) –2 (C) 2 (D) 4

Q.6 If $ax^2 + bx + c$, $a, b, c \in R$ has no real zeros and if $a + b + c < 0$ then

(A) $c > 0$ (B) $c < 0$
(C) $c = 0$ (D) None of these

Q.7 The integer k for which the inequality $x^2 - 2(4k - 1)x + (15k^2 - 2k - 7) > 0$ is valid for all real x is

(A) $3\frac{1}{2}$ (B) 5
(C) 3 (D) None of these

Q.8 The value of a for which the sum of the squares of the roots of equation $x^2 - (a - 2)x - (a + 1) = 0$ assumes least value is

(A) 1 (B) 0
(C) 2 (D) 3

Q.9 If both the roots of equation $(a - 3)x^2 - 2ax + 5a = 0$ are positive, then a lies in

(A) $[0, 15/4]$ (B) $]-\infty, 0[\ U\ ]3, \infty[$
(C) $[3, 15/4]$ (D) None of these

Q.10 If $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ have common root then numerical value of a + b is

(A) 2 (B) 1
(C) –1 (D) None of these

MATHEMATICS IIT JEE (JULY 4th WEEK CLASS TEST 1) (QUADRATIC EQUATION) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1					4					7				
2					5					8				
3					6					9				
										10				

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	C	B	D	C	A	B	B	A	C	C

SOLUTIONS

Sol.1 (C)

Let $f(x) = x^3 - 3x + 1$.

Check signs of $f(-2) = -1$, $f(-1) = +3$, $f(0) = 1$, $f(1) = -1$ & $f(2) = 3$.

Sign is changing 3 times in interval $[-2, 2]$.

Sol.2 (B)

$ax^2 + 2bx + c = 0$ has equal roots

$\Rightarrow b^2 = 4ac$

$ax^2 + 2bx + c + k$

$= a[\{x + b/a\}^2 + (ac - b^2)/a^2 + ka]$

$= a[(x + b/a)^2 + k/a]$

Which is +ve for every real x if $k > 0$, $a > 0$

Sol.3 (D)

let $\alpha' = \alpha/(\alpha - 1)$, $\beta' = \beta/(\beta - 1)$

$\Rightarrow \alpha = \alpha'/(\alpha' - 1)$, $\beta = \beta'/(\beta' - 1)$

$\Rightarrow 1/\alpha = (\alpha' - 1)/\alpha'$, $1/\beta = (\beta' - 1)/\beta'$

Then equation whose roots are $1/\alpha$, $1/\beta$, is $x^2 - (1/\alpha + 1/\beta)x + 1/\alpha\beta = 0$

$\Rightarrow x^2 - [(\alpha' - 1)/\alpha' + (\beta' - 1)/\beta']x + [(\alpha' - 1)(\beta' - 1)]/\alpha'\beta' = 0$

$\Rightarrow \alpha'\beta'x^2 - [2\alpha'\beta' - (\alpha' + \beta')]x + \alpha'\beta' - (\alpha' + \beta') + 1 = 0$

$\Rightarrow bx^2 - (a + 2b)x + a + b + 1 = 0$

Sol.4 (C)

$\alpha + \beta = 1/\alpha^2 + 1/\beta^2$

$\Rightarrow \alpha + \beta = [(\alpha + \beta)^2 - 2\alpha\beta]/(\alpha^2\beta^2)$

$\Rightarrow c^2/a^2 (-b/a) = b^2/a^2 - 2c/a$

$\Rightarrow -bc^2 = a(b^2 - 2ac)$

$\Rightarrow ab^2 + bc^2 = 2a^2c$

$\Rightarrow b/a = 2ac/(ab + c^2)$

$= [2(a/c)(c/b)]/(a/b + c/b)$

$\Rightarrow a/c, b/a, c/b$ are H. P.

Sol.5 (A)

$\alpha + \beta = a^2 - 2b$, $\alpha\beta = b^2$, $\alpha' + \beta' = b^2 - 2a$,

$\alpha'\beta' = a^2$, $\sqrt{\alpha} - \sqrt{\beta} = \sqrt{\alpha'} - \sqrt{\beta'}$

$\Rightarrow \alpha + \beta - 2\sqrt{(\alpha\beta)} = \alpha' + \beta' - 2\sqrt{(\alpha'\beta')}$

$\Rightarrow a^2 - 2b - 2b = b^2 - 2a - 2a$

$\Rightarrow a^2 - b^2 = -4(a - b)$

$\Rightarrow a + b = -4$

Sol.6 (B)

Since $ax^2 + bx + c = 0$ has no real zero

$\Rightarrow b^2 - 4ac < 0$ or $b^2 < 4ac$

$\Rightarrow 0 < b^2 < 4ac$

$\Rightarrow$ a and c will be of same sign.

Now $f(1) = a + b + c < 0$

as given $a + b + c < 0$

But $b^2 - 4ac < 0$

$\Rightarrow f(x) = ax^2 + bx + c$

will have same sign as that of a for all real x.

Since for $x = 1$, $f(1) < 0$

$\Rightarrow a < 0$

Since a, c have same sign $\Rightarrow c < 0$.

Sol.7 (B)

$f(x) = 1.x^2 - 2(4k - 1)x + (15k^2 - 2k - 7) > 0$ $\forall$ all real x

$\Rightarrow D = b^2 - 4ac \le 0$ and $a = 1 > 0$

$\Rightarrow 4(4k - 1)^2 - 4(15k^2 - 2k - 7) < 0$

$\Rightarrow (16k^2 - 8k + 1) - (15k^2 - 2k - 7) < 0$

$\Rightarrow k^2 - 6k + 8 < 0$

$\Rightarrow (k - 4)(k - 2) < 0$

$\Rightarrow k < 2$ or $4 < k$

Out of given options only $k = 5$ satisfies this condition.

Sol.8 (A)

Let α, β the roots then

$\alpha + \beta = (a - 2)$, $\alpha\beta = -(a + 1)$

Then $S = \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (a - 2)^2 + 2(a + 1)$

$= a^2 - 2a + 6$

$S = (a - 1)^2 + 5 \ge 5$

Sum of the squares of the roots will be least if $a - 1 = 0$

$\Rightarrow a = 1$

Sol.9 (C)

Both sum of the roots and products will positive and $D \geq 0$.

$\Rightarrow \alpha + \beta = \frac{2a}{a-3} > 0, \alpha\beta = \frac{5a}{a-3} > 0$

$4a^2 - 4.5a(a-3) \geq 0$

First two conditions give a > 3 or a < 0 and 3^{rd} condition gives

$a^2 - 5a^2 + 15a \geq 0$

$\Rightarrow a^2 - \frac{15}{4}a \leq 0$

$\Rightarrow a\left(a - \frac{15}{4}\right) \leq 0$

$\Rightarrow 0 \leq a \leq 15/4$

All these condition will be satisfied if

$3 \leq a \leq 15/4$

Sol.10 (C)

Given equations $x^2 + ax + b = 0$ ....(1)

$x^2 + bx + a = 0$ ....(2)

Subtracting we get $(a-b)x + (b-a) = 0$

$\Rightarrow$ x = 1 is the common root, which will satisfy the eq. (1) and (2), then on satisfying (1) we get

$1^2 + a.1 + b = 0$

$\Rightarrow a + b = -1$