Dear student following is an Easy level [●○○] test paper. Score of 24 Marks in 15 Minutes would be a satisfactory performance. Questions 1-10(+3,–1). (All questions have only one option correct).

Q.1 The condition that the equation

$$\frac{1}{x}+\frac{1}{x+b}=\frac{1}{m}+\frac{1}{m+b}$$

has real roots that are equal in magnitude but opposite in sign is

(A) $b^2 = m^2$ (B) $b^2 = 2m^2$

(C) $2b^2 = m^2$ (D) None of these

Q.2 Let a > 0, b > 0 and c > 0. Then both the roots of the equation $ax^2 + bx + c = 0$

(A) Are real and negative

(B) Have negative real parts

(C) Are rational numbers

(D) None of these

Q.3 Sum of the non-real roots of

$(x^2 + x - 2)(x^2 + x - 3) = 12$ ....(1) is

(A) 1 (B) –1

(C) –6 (D) 6

Q.4 Find the sum all the real numbers satisfying the equation $x^2 + |x - 1| = 1$

(A) 1 (B) 0

(C) 2 (D) 3

Q.5 Find the number of polynomials p(x) with integral coefficients satisfying the conditions

$p(1) = 2,\ p(3) = 1.$

(A) 0 (B) 1

(C) 2 (D) 3

Q.6 The number of irrational roots of the equation $\frac{4x}{x^2+x+3}+\frac{5x}{x^2-5x+3}=-\frac{3}{2}$ is

(A) 0 (B) 1 (C) 2 (D) 3

Q.7 The number of real values of x which satisfy the equation $\left|\frac{x}{x-1}\right| + |x| = \frac{x^2}{|x-1|}$

(A) 1 (B) 2 (C) 5 (D) Infinite

Q.8 In a triangle PQR, $\angle R = \pi/2$. If tan (P/2) and tan (Q/2) are the roots of the equation $ax^2 + bx + c = 0$ where $a \neq 0$, then

(A) a + b = c (B) b + c = a

(C) a + c = b (D) b = c

Q.9 If b > a, then the equation

$(x - a)(x - b) - 1 = 0$ has

(A) Both roots in [a, b]

(B) Both roots in $(-\infty, a)$

(C) Both roots in (b, ∞)

(D) One root in $(-\infty, a)$ and other in (b, ∞)

Q.10 Suppose p, q, r, s $\in$ R and α, β be the roots of $x^2 + px + q = 0$ and α^4, β^4 be the roots of $x^2 - rx + s = 0$, then the equation

$x^2 - 4qx + 2q^2 - r = 0$ has always

(A) Two imaginary roots

(B) Two positive roots

(C) Two negative roots

(D) One positive and one negative root

MATHEMATICS IIT JEE (JULY 4th WEEK CLASS TEST 2) (QUADRATIC EQUATION) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○
										10	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	B	B	B	A	A	C	D	A	D	D

SOLUTIONS

Sol.1 (B)

Clearly x = m is a root of the equation.

Therefore, the other root must be – m.

i.e., $\frac{1}{-m}+\frac{1}{-m+b} = \frac{1}{m} + \frac{1}{m+b}$

$\Rightarrow \frac{1}{b-m} - \frac{1}{m+b} = \frac{2}{m}$

$\Rightarrow \frac{b+m-b+m}{b^2-m^2} = \frac{2}{m}$

$\Rightarrow 2m^2 = 2b^2 - 2m^2$ or $2m^2 = b^2$.

Sol.2 (B)

We have $D = b^2 - 4ac$. If $D \geq 0$, then the roots of the equation are given by

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

As $D = b^2 - 4ac < b^2$ ($\because a > 0, c > 0$), it follows that the roots of quadratic equation are negative. In case $D < 0$, then the roots of the equation are given by

$$x = \frac{-b \pm i\sqrt{-D}}{2a}$$

Which have negative real parts.

Sol.3 (B)

Put $x^2 + x = y$. So that the given equation becomes

$(y - 2)(y - 3) = 12$

$\Rightarrow y^2 - 5y - 6 = 0$

$\Rightarrow (y - 6)(y + 1) = 0 \Rightarrow y = 6, -1.$

When y = 6, we get $x^2 + x - 6 = 0$

$\Rightarrow (x + 3)(x - 2) = 0$ or $x = -3, 2$

When y = –1, we get $x^2 + x + 1 = 0$

$\Rightarrow x = \omega, \omega^2$ and their sum is –1.

Sol.4 (A)

For $x \geq 1$, given equation becomes

$x^2 + x - 1 = 1 \Leftrightarrow (x + 2)(x - 1) = 0$

$\Rightarrow x = 1$

For $x < 1$, given equation becomes

$x^2 + 1 - x = 1 \Rightarrow x = 0$

Sol.5 (A)

Let p(x) be one such polynomial.

Let $p_1(x)$, be such that

$p(x) - 1 = (x - 3)\, p_1(x)$

$\Rightarrow p(1) - 1 = 2p_1(1)$

$\Rightarrow 1 = -2p_1(1)$

But this is not possible as both 2 and $p_1(1)$ are integers.

Sol.6 (C)

x = 0 is not a root. Divide both the numerators and denominators by x and put x + 3/x = y to obtain

$\frac{4}{y+1} + \frac{5}{y-5} = -\frac{3}{2} \Rightarrow y = -5, 3$

x + 3/x = –5 has two irrational roots and

x + 3/x = 3 has imaginary roots.

Sol.7 (D)

Since $\left|\frac{x}{x-1}+x\right| = \frac{x^2}{|x-1|}$, we must have x

and $\frac{x}{x-1}$ are of the same sign i.e. $\frac{x^2}{x-1} \geq 0$

$\Rightarrow x = 0$ or $x - 1 > 0$

Sol.8 (A)

We have

$$\tan\left(\frac{P}{2}\right) + \tan\left(\frac{Q}{2}\right) = \frac{-b}{a}$$

and $\tan\left(\frac{P}{2}\right)\tan\left(\frac{Q}{2}\right) = \frac{c}{a}$

Now, $P + Q = \frac{\pi}{2} \Rightarrow 1 = \tan\left(\frac{\pi}{4}\right)$

$= \tan\left(\frac{P}{2}+\frac{Q}{2}\right) = \frac{\tan(p/2)+\tan(Q/2)}{1-\tan(P/2)\tan(Q/2)}$

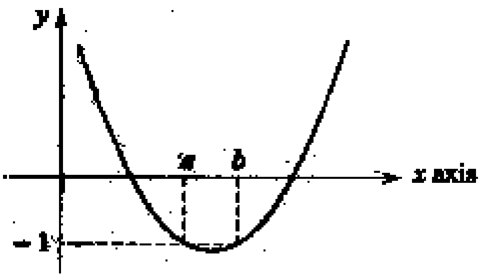
$\Rightarrow 1 = \frac{-b/a}{1-c/a} = \frac{-b}{a-c}$

$\Rightarrow a - c = -b$

$\Rightarrow c = a + b$

Sol.9 (D)

Graph of $y = (x - a)(x - b) - 1$ is given as



It is a parabola which open upwards. Also, $y < 0$ for $x = a$ and $x = b$.

$\therefore$ $y = (x - a)(x - b) - 1$ meets the x-axis at two points once in $(-\infty, a)$ and once in (b, ∞). Thus. one root lies in $(-\infty, a)$ and one in (b, ∞).

Sol.10 (D)

We have

$\alpha + \beta = -p, \alpha\beta = q, \alpha^4 + \beta^4 = r, \ \alpha^4\beta^4 = s$

Discriminant D of the equation

$x^2 - 4qx + 2q^2 - r = 0$ .....(1)

is given

$D = 16q^2 - 4(2q^2 - r) = 8q^2 + 4r = 4[2q^2 + r]$

$= 4[2\alpha^2\beta^2 + \alpha^4 + \beta^4] = 4(\alpha^2 + \beta^2)^2 \geq 0$

Thus, the equation (1) has real roots.

The roots of (1) are given by

$$x = \frac{4q \pm \sqrt{D}}{2} = 2\alpha\beta \pm (\alpha^2 + \beta^2)$$

$$= (\alpha + \beta)^2, -(\alpha - \beta)^2$$

Hence, (1) has one positive and one negative root.