Dear student following is a Moderate level [○●○] test paper. Score of 18 Marks in 15 Minutes would be a satisfactory performance. Questions 1-10(+3,–1). (All questions have only one option correct).

Q.1 Let P(x) be a polynomial with integral coefficients. If there exist two integers a and b such that P(a) – P(b) = 1 then,
(A) Both a and b must be even
(B) Both a and b must be old
(C) a and b are two consecutive integers
(D) None of these

Q.2 If $0 < a < b < c < d$, then the quadratic equation $ax^2 + \{1 - a(b + c)\}x + abc - d = 0$ ...(1) has
(A) Real and distinct roots out of which one lies between c and d.
(B) Real and distinct roots out of which one lies between a and b
(C) Real and distinct roost out of which one lies between b and c
(D) Non-real roots

Q.3 Let $f(x) = ax^2 + bx + c$, a, b, c $\in$ R and a $\neq$ 0. Suppose f(x) > 0 for all real x and let g(x) = f(x) = f(x) + f'(x) + f"(x), then
(A) $g(x) > 0\ \forall x \in R$ (B) $g(x) < 0\ \forall x \in R$
(C) g(x) = 0 has real roots
(D) None of these

Q.4 If the roots equation $x^2 - 2ax + (a^2 + a - 3) = 0$ be each less then 3, then
(A) a < 2 (B) $2 \le a \le 3$
(C) $3 < a \le 4$ (D) a > 4

Q.5 The number of quadratic equation which remain unchanged on squaring their roots is
(A) 1 (B) 2
(C) 4 (D) None of these

Q.6 If 0 < a < b < c, and the roots α and β of the equation $ax^2 + bx + c = 0$ are imaginary then
(A) $|\alpha| > |\beta|$ (B) $|\alpha| < |\beta|$
(C) $|\alpha| > 1$ (D) $|\beta| < 1$

Q.7 Let S be the set of values of 'a' for which 2 lie between the roots of the quadratic equation $x^2 + (a + 2)x - (a + 3) = 0$ then S is given by
(A) $(-\infty, -5)$ (B) $(5, \infty)$
(C) $(-\infty, -5]$ (D) $[5, \infty)$

Q.8 If p, q, r are positive and are in A.P., the roots of the quadratic equation $px^2 + qx + r = 0$ are real for
(A) $\left|\frac{r}{p} - 7\right| \ge 4\sqrt{3}$ (B) $\left|\frac{p}{r} - 7\right| < 4\sqrt{3}$
(C) All p and all r (D) No p and no r.

Q.9 If the expression $\left(mx - 1 + \frac{1}{x}\right)$ is non-negative for all positive real x, then the minimum value of m must be
(A) –1/2 (B) 0
(C) 1/4 (D) 1/2

Q.10 Let p(x) = 0 be a polynomial equation of least possible degree, with rational coefficients, having $\sqrt[3]{7} + \sqrt[3]{49}$ as one of its roots. Then the product of all the roots of p(x) = 0 is
(A) 7 (B) 49
(C) 56 (D) 63

MATHEMATICS IIT JEE (JULY 4th WEEK CLASS TEST 3) (QUADRATIC EQUATION) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○
										10	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	C	A	A	A	C	C	A	A	C	C

SOLUTIONS

Sol.1 (C)

Let $P(x) = a_0 x^n + a_1 x^{n-1} + \ldots\ldots + a_{n-1} x + a_n$ where $a_0, a_0, \ldots\ldots, a_n$ are integers

Now, $1 = P(a) - P(b) = a_0(a^n - b^n) + a_1(a^{n-1} - b^{n-1}) + a_2(a^{n-2} - b^{n-2}) + \ldots + a_{n-1}(a-b)$
$= (a - b)m$

Where $m = a_0(a^{n-1} + a^{n-2}b + \ldots + ab^{n-2} + b^{n-1}) + a_1(a^{n-2} + a^{n-3}b + \ldots + ab^{n-3} + b^{n-2}) + \ldots + a_{n-2}(a + b) + a_{n-1}$

is an integer.

As product of two integers is 1, either $a - b = 1$ and $m = 1$ or $a - b = -1$ and $m = -1$.

$\therefore$ a and b must be two consecutive integers.

Sol.2 (A)

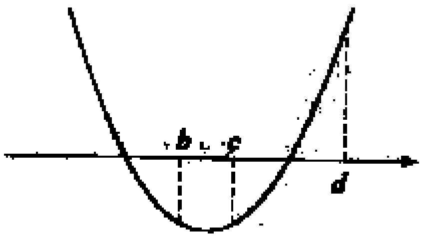
We can rewrite (1) as

$ax^2 - a(b + c)x + abc + x - d = 0$

or $a(x - b)(x - c) + x - d = 0$

Let $f(x) = a(x - b)(x - c) + x - d$.

As $a > 0$, $y = f(x)$ represent a parabola which open upwards. See



Also, $f(b) = b - d < 0$
$f(c) = c - d < 0$,
and $f(d) = a(d - b)(d - c) > 0$.

Thus $f(x) = 0$ has root between $-\infty$ and b and between c and d.

Sol.3 (A)

Given function $g(x) = f(x) + f'(x) + f''(x)$

$= ax^2 + bx + c + 2ax + b + 2a$

$= ax^2 + (2a + b)x + (2a + b + c)$

$\because$ f(x) is positive for all real x

$\Rightarrow$ $a > 0$ and $D_1 = b^2 - 4ac < 0$

Now, the D of g(x) is

$D = (2a + b)^2 - 4a(2a + b + c)$
$= 4a^2 + b^2 + 4ab - 8a^2 - 4ab - 4ac$
$= b^2 - 4a^2 - 4ac$
$= b^2 - 4ac - 4a^2 = D_1 - 4a^2 < 0$ as $D_1 < 0$

Thus D is – ive and a is + ive

$\therefore$ g(x) is also + ive for all real x.

Sol.4 (A)

Let $f(x) \equiv x^2 - 2ax + (a^2 + a - 3) = 0$

$\because$ Roots are each less then 3, then

$f(3) > 0$ and $D = B^2 - 4AC \geq 0$

Now $f(3) = 9 - 6a + a^2 + a - 3 > 0$

$\Rightarrow$ $a^2 - 5a + 6 > 0$

$\Rightarrow$ $(a - 3)(a - 2) > 0$

$\Rightarrow$ $a < 2$ or $3 < a$ .....(1)

And $D = (-2a)^2 - 4(a^2 + a - 3) \geq 0$

$\Rightarrow$ $a \leq 3$ ....(2)

Both conditions (1) and (2) will be satisfied if $a < 2$.

Sol.5 (C)

Let α, β be the roots of the equation such an equation will remain unchanged on squaring the roots if $\alpha^2 + \beta^2 = \alpha + \beta$
....(1)

$\alpha^2 \beta^2 = \alpha\beta$ ....(2)

from (2) $\alpha\beta(\alpha\beta - 1) = 0$

$\Rightarrow$ $\alpha\beta = 0$ or $\alpha\beta = 1$

If $\alpha = 0$ then eq. (1) gives $\beta(\beta - 1) = 0$

$\Rightarrow$ $\beta = 0, 1$

$\Rightarrow$ roots will be 0, 0 or 0, 1 similarly if we take $\beta = 0$ we get same pairs of roots.

If $\alpha\beta = 1$ $\Rightarrow$ $\beta = 1/\alpha$ then eq. (1) gives

$$\alpha^2 + \frac{1}{\alpha^2} = \alpha + \frac{1}{\alpha} \Rightarrow \alpha^4 + 1 = \alpha^3 + \alpha$$

$\Rightarrow$ $\alpha^4 - \alpha^3 - \alpha + 1 = 0$

$\Rightarrow$ $(\alpha^3 - 1)(\alpha - 1) = 0$

$\Rightarrow$ $\alpha^3 = 1$ or $\alpha = 1$

$\Rightarrow$ $\alpha = 1, \omega, \omega^2, 1$

If $\alpha = 1$ we get $\beta = \frac{1}{\alpha} = 1$

and if $\alpha = \omega$ we get $\beta = \frac{1}{\alpha} = \frac{1}{\omega} = \omega^2$

and if $\alpha = \omega^2$ we get $\beta = \frac{1}{\omega^2} = \alpha$

Thus roots are, 1 ,1 or ω, ω^2.

Thus their are four such equations whose roots are, 0, 0 and 0, 1 and 1, 1 and ω, ω^2.

Sol.6 (C)

Since 0 < a < b < c then a, b, c are real and then imaginary roots α and β will form a conjugate pair.

$\Rightarrow \beta = \bar{\alpha} \Rightarrow |\beta| = |\bar{\alpha}| = |\alpha|$

$\Rightarrow$ Option (a) and (b) are wrong.

Now products of the roots is

$$\alpha\beta = \frac{c}{\alpha} \Rightarrow \alpha\bar{\alpha} = \frac{c}{\alpha} > 1 \quad [\because \ c > a > 0]$$

$$\Rightarrow |\alpha^2| = \frac{c}{a} \Rightarrow |\beta| = |\alpha| = \sqrt{\frac{c}{a}} > 1.$$

Sol.7 (A)

Let $f(x) \equiv x^2 + (a + 2).\, x - (a + 3) = 0$

Since 2 lies between the roots that also means the roots are real

$\Rightarrow f(2) = 2^2 + (a + 2)2 - (a + 3) < 0$

or $5 + a < 0 \Rightarrow a < -5$

And $D = B^2 - 4AC = (a + 2)^2 + 4(a + 3)$,

$= (a + 4)^2 \geq 0$ for all α.

Thus set $S = (-\infty, -5)$.

Sol.8 (A)

Since p, q and r are in A.P., 2q = p + r

The roots of $px^2 + qx + r = 0$ are real if

$$q^2 - 4pr \geq 0 \Rightarrow \left(\frac{p+r}{2}\right)^2 - 4pr \geq 0$$

$$\Rightarrow p^2 + r^2 - 14pr \geq 0$$

$$\Rightarrow \left(\frac{r}{p}\right)^2 - 14\left(\frac{r}{p}\right) + 1 \geq 0$$

$$\text{Now, } \left(\frac{r}{p}\right)^2 - 14\left(\frac{r}{p}\right) + 1 = 0$$

$$\Rightarrow \left(\frac{r}{p}\right) = 7 \pm 4\sqrt{3}.$$

$$\text{So, } \left(\frac{r}{p}\right)^2 - 14\left(\frac{r}{p}\right) + 1 \geq 0$$

$$\Rightarrow \left(\frac{r}{p}\right) \leq 7 - 4\sqrt{3} \text{ or } \frac{r}{p} \geq 7 + 4\sqrt{3}.$$

$$\Rightarrow \frac{r}{p} - 7 \leq -4\sqrt{3} \quad \text{or } \frac{r}{p} - 7 \geq 4\sqrt{3}.$$

$$\Rightarrow \left|\frac{r}{p} - 7\right| \geq 4\sqrt{3}.$$

Hence (A) is the correct answer

Sol.9 (C)

We know that $ax^2 + bx + c \geq 0$ if $a > 0$ and $b^2 - 4ac \leq 0$.

$$\text{So, } mx - 1 + \frac{1}{x} \geq 0 \Rightarrow \frac{mx^2 - x + 1}{x} \geq 0$$

$\Rightarrow mx^2 - x + 1 \geq 0$ as $x > 0$

Now, $mx^2 - x + 1 \geq 0$ if $m > 0$ and $1 - 4m \leq 0$ or if $m > 0$ and $m \geq 1/4$.

Thus, the minimum value of m is 1/4.

Sol.10 (C)

$$x = \sqrt[3]{7} + \sqrt[3]{49}$$

$$\Rightarrow x^3 = 7 + 49 + 3\sqrt[3]{7}\,.\,\sqrt[3]{49}\left(\sqrt[3]{7} + \sqrt[3]{49}\right)$$

$$\Rightarrow x^3 - 21x - 56 = 0$$

$\Rightarrow$ Product of roots = 56.