

Dear student following is a Moderate level [O ● O] test paper. Score of 18 Marks in 15 Minutes would be a satisfactory performance. Questions 1-10 (+3, -1). (Questions may have more than one option correct)

- Q.1** The set of values of 'a' for which the equation $x^3 - 3x + a = 0$ has three distinct real roots, is
 (A) $(-\infty, \infty)$ (B) $(-2, 2)$
 (C) $(-1, 1)$ (D) None of these
- Q.2** Consider the equation $x^2 + x - n = 0$, where n is an integer lying between 1 to 100. Total number of different values of 'n' so that the equation has integral roots is
 (A) 6 (B) 4
 (C) 9 (D) None
- Q.3** If a, b, c are in G.P., then the equation $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root if $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in
 (A) A.P. (B) G.P. (C) H.P. (D) None
- Q.4** If both the roots of the equation $x^2 - (p - 4)x + 2e^{2 \ln p} - 4 = 0$ are negative then p belongs to
 (A) $(-\sqrt{2}, 4)$ (B) $(\sqrt{2}, 4)$
 (C) $(-4, \sqrt{2})$ (D) $(-\infty, -\sqrt{2})$
- Q.5** If $x^2 - 4x + \log_{1/2} a = 0$ does not have two distinct real roots, then maximum value of a is
 (A) $\frac{1}{4}$ (B) $\frac{1}{16}$
 (C) $-\frac{1}{4}$ (D) None
- Q.6** If the roots of the equation $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$ be equal then a, b, c are in
 (A) H.P. (B) G.P. (C) A.P. (D) None
- Q.7** All the values of a for which the equation $2x^2 - 2(2a + 1)x + a(a - 1) = 0$ has roots α and β satisfying the condition $\alpha < a < \beta$
 (A) $a \in (-\infty, -3) \cup (0, \infty)$
 (B) $a \in (0, \infty)$
 (C) $a \in (-\infty, -3)$
 (D) None of these
- Q.8** If x is real, then the minimum value of $\frac{(a+x)(b+x)}{(c+x)}$ ($x > -c$), for $a > c, b > c$ is
 (A) $(\sqrt{a+c} + \sqrt{b+c})^2$ (B) $(\sqrt{a-c} + \sqrt{b-c})^2$
 (C) $(\sqrt{a+c} - \sqrt{b-c})^2$ (D) $(\sqrt{a-c} - \sqrt{b-c})^2$
- Q.9** If $2a + 3b + 6c = 0$ ($a, b, c \in \mathbb{R}$) then the equation $ax^2 + bx + c = 0$ has atleast one root between
 (A) 0 and 2 (B) 0 and 1
 (C) -1 and 0 (D) None
- Q.10** If $a < b < c < d$, then the equation $3(x - a)(x - c) + 5(x - b)(x - d) = 0$ has
 (A) real and distinct roots
 (B) real and equal roots
 (C) purely imaginary roots
 (D) None of these

MATHEMATICS IIT JEE (JULY 4th WEEK CLASS TEST 4) (QUADRATIC EQUATION) ANSWER KEY

Name : Roll No. :

	A	B	C	D		A	B	C	D		A	B	C	D
1	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	5	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	9	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
2	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	6	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	10	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
3	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	7	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>					
4	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	8	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>					

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	B	A	A	B	B	A	A	B	A,B	A

SOLUTIONS

Sol.1 (B)

Let $f(x) = x^3 - 3x + a$

$f'(x) = 3x^2 - 3$

For three distinct real roots

(i) $f'(x) = 0$ should have two distinct real roots α and β

and (ii) $f(\alpha) f(\beta) < 0$

Here $\alpha = 1, \beta = -1$. Now $f(\alpha) f(\beta) < 0$

$\Rightarrow (1 - 3 + a)(-1 + 3 + a) < 0$

$\Rightarrow (a - 2)(a + 2) < 0$

$\Rightarrow -2 < a < 2$.

Sol.2 (A)

$x^2 + x - n = 0$, discriminant = $1 + 4n =$ odd number = D(say)

Now given equation would have a integral solution if D is a perfect square.

Let $D = (2\lambda + 1)^2$

$\Rightarrow n = \lambda + \lambda^2$

$\Rightarrow n = \lambda(\lambda + 1) =$ even number

$\Rightarrow n$ can be 2, 6, 2, 20, 30, 42, 56, 72, 90.

Sol.3 (A)

a, b, c are in G.P.

$\Rightarrow b^2 = ac$

Now the equation $ax^2 + 2bx + c = 0$ can be rewritten as

$ax^2 + 2\sqrt{ac}x + c = 0$

$\Rightarrow (\sqrt{a}x + \sqrt{c})^2 = 0$

$\Rightarrow x = -\sqrt{\frac{c}{a}}, -\sqrt{\frac{c}{a}}$

If the two given equations have a common

root, then this root must be $-\sqrt{\frac{c}{a}}$.

Thus $d\frac{c}{a} - 2e\sqrt{\frac{c}{a}} + f = 0$

$\Rightarrow \frac{d}{a} + \frac{f}{c} = \frac{2e}{c}\sqrt{\frac{c}{a}} = \frac{2e}{\sqrt{ac}} = \frac{2e}{b}$

$\Rightarrow \frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

Sol.4 (B)

Both root negative

\Rightarrow sum of roots < 0 and product of roots > 0

$\Rightarrow p - 4 < 0$ and $2e^{2 \ln p} - 4 > 0$

$\Rightarrow p < 4$ and $p^2 > 2$

$\Rightarrow p \in (\sqrt{2}, 4)$

Sol.5 (B)

Since $x^2 - 4x + \log_{1/2} a = 0$ does not have two distinct real roots, discriminant ≤ 0

$\Rightarrow 16 - 4 \log_{1/2} a \leq 0$

$\Rightarrow \log_{1/2} a \geq 4$

$\Rightarrow a \leq \frac{1}{16}$

Sol.6 (A)

Given

$a(b - c)x^2 + b(c - a)x + c(a - b) = 0 \dots(i)$

let $A = a(b - c), B = b(c - a), C = c(a - b)$

$\therefore A + B + C = 0 \dots(ii)$

since the roots of the equation (i) are equal

$\therefore B^2 - 4AC = 0$

$(-A - C)^2 - 4AC = 0$ (using (ii))

$(A + C)^2 - 4AC = 0$

$(A - C)^2 = 0$

$A = C$

$\Rightarrow a(b - c) = c(a - b)$

$\Rightarrow 2ac = b(a + c)$

$\Rightarrow b = \frac{2ac}{a+c}$

$\Rightarrow a, b, c$ are in H.P.

Sol.7 (A)

\therefore Number a lies between the roots of the given equation then

$2f(a) < 0$

$f(a) < 0$

$\therefore 2a^2 - 2(2a + 1)a + a(a - 1) < 0$

$\Rightarrow -a^2 - 3a < 0$

$\Rightarrow a(a + 3) > 0$

Thus from wavy curve method 

$a \in (-\infty, -3) \cup (0, \infty)$

Sol.8 (B)

Let $y = \frac{(a+x)(b+x)}{(c+x)}$

$\Rightarrow x^2 + (a+b)x + ab = cy + xy$

$\Rightarrow x^2 + (a+b-y)x + ab - cy = 0$

For real x $B^2 - 4AC \geq 0$

$\Rightarrow (a+b-y)^2 - 4ab + 4cy \geq 0$

$\Rightarrow (a-b)^2 + y^2 - 2(a+b)y - 4ab + 4cy \geq 0$

$\Rightarrow (a-b)^2 + y^2 - 2(a+b-2c)y \geq 0$

$\Rightarrow y^2 - 2(a+b-2c)y + (a-b)^2 \geq 0$

$\Rightarrow [y^2 - (\sqrt{a-c} - \sqrt{b-c})^2][y - (\sqrt{a-c} + \sqrt{b-c})^2] \geq 0$

$\therefore y \leq (\sqrt{a-c} - \sqrt{b-c})^2$ and $y \geq (\sqrt{a-c} + \sqrt{b-c})^2$

Hence minimum value of y is

$$(\sqrt{a-c} + \sqrt{b-c})^2$$

Sol.9 (A, B)

Given $2a + 3b + 6c = 0$

Let $f(x) = ax^2 + bx + c$, then

$$f(x) = \frac{ax^3}{3} + \frac{bx^2}{2} + cx + k$$

$$= \frac{2ax^3 + 3bx^2 + 6cx + 6k}{6}$$

Now $f(1) = \frac{2a + 3b + 6c + 6k}{6}$

$$= \frac{6k}{6} = k \quad [\because 2a + 3b + 6c = 0]$$

Also $f(0) = \frac{6k}{6} = k$

Thus $f(0) = f(1)$, therefore equation $f'(x) = 0$ i.e. equation $ax^2 + bx + c = 0$ will have at least one real root between 0 and 1 and hence equation $ax^2 + bx + c = 0$ will have at least one root between 0 and 2.

Sol.10 (A)

Let $f(x) = 3(x-a)(x-c) + 5(x-b)(x-d)$

Since f is a polynomial f is continuous on R.

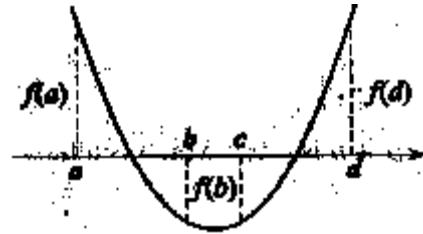
Also, since $a < b < c < d$,

$$f(a) = 5(a-b)(a-d) > 0$$

$$f(b) = 3(b-a)(b-c) < 0$$

$$f(c) = 5(c-b)(c-d) < 0$$

$$f(d) = 3(d-a)(d-c) > 0$$



As f is continuous on R, $y = f(x)$ crosses x-axis at least once between a and b and once between c and d. See fig.

Thus, f has two distinct real roots, one lying between a and b, one between c and d.