Dear student following is a Moderate level [O ● O O O] test paper. Score of 18 Marks in 15 Minutes would be a satisfactory performance. Questions 1-10 (+3, –1). (All Questions have only Option correct)

Q.1 Let a, b, c be real and $ax^2 + bx + c = 0$ has two real roots α and β where $\alpha < -1$ and $\beta > 1$, then $1 + \frac{c}{a} + \left|\frac{b}{a}\right|$

(A) < 2 (B) < 1 (C) < 0 (D) None

Q.2 If one root of the equation $x^2 + px + 12 = 0$ is 4, while the equation $x^2 + px + q = 0$ has equal roots, the value of q is

(A) $\frac{49}{4}$ (B) $\frac{4}{49}$

(C) 4 (D) None

Q.3 If the expression $y^2 + 2xy + 2x + my - 3$ can be resolved into two rational factors, then m must be

(A) Any +ve real no. (B) Any –ve real no.

(C) –2 (D) 3

Q.4 The roots of an equation

$$x^3 - 9x^2 + 14x + 24 = 0$$

are in the ratio 3 : 2. The roots will be

(A) 6, 4, –1 (B) 6, 4, 1

(C) –6, 4, 1 (D) –6, –4,1

Q.5 Let α, β be the roots of the quadratic equation $x^2 + px + p^3 = 0$ $(p \neq 0)$. If (α, β) is a point on the parabola $y^2 = x$, then the roots of the quadratic equation are

(A) 4, –2 (B) –4, –2

(C) 4, 2 (D) –4, 2

Q.6 If $a, b, c \in R$ and equations $ax^2 + bx + c = 0$ and $x^2 + 2x + 9 = 0$ have a common root, then a : b : c =

(A) 1 : 2 : 9 (B) 2 : 9 : 1

(C) 9 : 2 : 1 (D) None of these

Q.7 Let $f(x) = x^2 - ax + b$, a is odd +ve integer f(x) = 0 have two prime numbers as roots and a + b = 23. Then the value of f(1) + f(2) + f(3) + ... + f(10) is

(A) 30 (B) 100

(C) 120 (D) 80

Q.8 The equation $x^2 + nx + m = 0$, $n, m \in I$ cannot have

(A) Integral roots (B) non integral roots

(C) Irrational roots (D) Complex roots

Q.9 If a, b, c be the sides of ΔABC and equations $ax^2 + bx + c = 0$ and $5x^2 + 12x + 13 = 0$ have a common root, then $\angle C$ is

(A) 60º (B) 90º

(C) 120º (D) 45º

Q.10 If $ax^2 + bx + c = 0$ and $cx^2 + bx + a = 0$ $(a, b, c \in R)$ have a common non real root, then

(A) $-2|a| < 5b < 2|a|$

(B) $-2|c| < b < 2|c|$

(C) $a \neq c$

(D) None

MATHEMATICS IIT JEE (JULY 5th WEEK CLASS TEST 1) (QUADRATIC EQUATION) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	⬭	⬭	⬭	⬭	5	⬭	⬭	⬭	⬭	9	⬭	⬭	⬭	⬭
2	⬭	⬭	⬭	⬭	6	⬭	⬭	⬭	⬭	10	⬭	⬭	⬭	⬭
3	⬭	⬭	⬭	⬭	7	⬭	⬭	⬭	⬭					
4	⬭	⬭	⬭	⬭	8	⬭	⬭	⬭	⬭					

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	C	A	C	A	A	A	A	B	B	B

SOLUTIONS

Sol.1 (C)

$\because$ $\alpha < -1$ and $\beta > 1$

$\therefore$ $\alpha + \lambda = -1$ and $\beta = 1 + \mu$ where $\lambda, \mu > 0$

Now $1 + \frac{c}{a} + \left|\frac{b}{a}\right| = 1 + \alpha\beta + |\alpha + \beta|$

$= 1 + (-1 - \lambda)(1 + \mu) + |-1 - \lambda + 1 + \mu|$

$= -\mu - \lambda - \lambda\mu + |\mu - \lambda|$

$$= \begin{cases} -\mu - \lambda - \lambda\mu + \mu - \lambda, & \mu > \lambda \\ -\mu - \lambda - \lambda\mu - \mu + \lambda, & \mu < \lambda \end{cases}$$

$\Rightarrow$ $1 + \frac{c}{a} + \left|\frac{b}{a}\right| = -2\lambda - \lambda\mu$ or $-2\mu - \lambda\mu$

Thus in both the case

$1 + \frac{c}{a} + \left|\frac{b}{a}\right| < 0$

As, $\mu, \lambda > 0$.

Sol.2 (A)

$\because$ 4 is a root of $x^2 + px + 12 = 0$

we have $16 + 4p + 12 = 0$

$\Rightarrow$ $p = -7$

Also given roots of $x^2 + px + q = 0$ are equal

$\Rightarrow$ $p^2 - 4q = 0$

$\Rightarrow$ $49 - 4q = 0$

$\Rightarrow$ $q = \frac{49}{4}$

Sol.3 (C)

Given equation can be rewritten as

$y^2 + y(2x + m) + 2x - 3 = 0$

$\therefore$ $D = (2x + m)^2 - 4(2x - 3)$

$D = 4x^2 + 4xm - 8x + m^2 + 12$

$D = 4x^2 + 4x(m - 2) + m^2 + 12$

$D = 4x^2 + 4x(m - 2) + (m - 2)^2 + m^2 + 12 + (m - 2)^2$

$D = [2x + (m - 2)]^2 + [8 + 4m]$

As we know that for rational factors, D should bea perfect square, implies

$8 + 4m = 0$

$\Rightarrow$ $m = -2$.

Sol.4 (A)

Since sum of three roots 6, 4, –1 is equal to 9.

$\therefore$ given equation has no real root

Sol.5 (A)

Since α, β are roots of the equation

$x^2 + px + p^3 = 0$

$\therefore$ $\alpha + \beta = -p$ .....(1)

and $\alpha\beta = p^3$ .....(2)

Since (α, β) lies on parabola $y^2 = x$

$\therefore$ $\beta^2 = \alpha$ .....(3)

From (2) and (3), we get

$\beta^2 = p^3$ or $\beta = p$.

From (1) and (3), we get

$\beta^2 + \beta = -p$

$\therefore$ $p^2 + p = -p$ $(\because p \neq 0)$

$\Rightarrow$ $p(p + 2) = 0$

$\Rightarrow$ $p = -2$

$\therefore$ $\alpha = 4, \beta = -2$.

Sol.6 (A)

Given equations are

$ax^2 + bx + c = 0$ ......(1)

and $x^2 + 2x + 9 = 0$ ......(2)

clearly roots of equation (1) are imaginary since equation (1) and (2) have a common root, therefore common root must be imaginary and hence both roots will be common. Thus equation (1) and (2) are identical.

$\therefore$ $\frac{a}{1} = \frac{b}{2} = \frac{c}{9}$

$\Rightarrow$ $a : b : c = 1 : 2 : 9$

Sol.7 (A)

$b + a$ = sum of roots = odd (given)

But roots are prime. Both cannot be odd as their sum is odd.

$\therefore$ 2 is one root

$\Rightarrow$ $a + b = 23$ .....(1)

and $4 - 2a + b = 0$ .....(2)

On solving (1) & (2) we get $a = 9$ and $b = 14$

$\therefore$ $f(1) = 1 - 9 + 14$

$f(2) = 4 - 18 + 14$

$\vdots$

$f(10) = 100 - 90 + 14$

$\Rightarrow$ $f(1) + f(2) + f(3) + ... + f(10) = 30$

Sol.8 (B)

The equation can be written as

$$x(x+ n) + m = 0$$

If x is a non integral rational no., then both x and (x + n) will have same denominator (±1) and x(x + n) will not be an integer.

The sum of a non integer and an integer can never be zero.

$\therefore$ The given equation cannot have non integral rational roots.

Sol.9 (B)

$\because$ $5x^2 + 12x + 13 = 0$

has imaginary roots as

$$D = 144 - 4 \times 5 \times 13 = -116 < 0$$

given, both roots of $ax^2 + bx + c = 0$ and $5x^2 + 12x + 13 = 0$ are common, implies both the equations are identical

$$\therefore \quad \frac{a}{5} = \frac{b}{12} = \frac{c}{13}$$

$\Rightarrow$ $a^2 + b^2 = c^2$

$\Rightarrow$ $\angle C = 90^\circ$

Sol.10 (B)

Given roots of both the equations are non real, implies

$$D_1 = b^2 - 4ac < 0 \text{ and } D_2 = b^2 - 4ac < 0$$

and also both roots will be common if

$$\frac{a}{c} = \frac{b}{b} = \frac{c}{a} = 1$$

$\Rightarrow$ $a = c$

Now $b^2 - 4ac < 0$

$\Rightarrow$ $b^2 - 4a^2 < 0$ or $b^2 - 4c^2 < 0$

$\Rightarrow$ $|b| < 2|a|$ or $|b| < 2|c|$