Dear student following is a Moderate level [O ● O] test paper. Score of 18 Marks in 15 Minutes would be a satisfactory performance. Questions 1-10 (+3, –1). (All Questions have only one option correct)

PASSAGE I (Q.NO. 1 - 4) :

There is a quadratic equation $ax^2 + bx + c = 0$. Whose smaller root is 'α' and bigger root is 'β'.

Q.1 a . c < 0, then :
(A) Roots are equal in magnitude but opposite in sign.
(B) Roots are opposite in sign.
(C) Roots are having same sign
(D) Roots are having same sign and equal magnitude

Q.2 a > 0, b > 0, c > 0, then-
(A) Roots are negative
(B) Roots are positive
(C) One root is negative and another root is positive
(D) Nothing can be said

Q.3 Sign of a = sign of b $\neq$ sign of c, then-
(A) $\alpha + \beta < 0$ (B) $0 < \alpha < \beta$
(C) $\alpha < 0 < |\alpha| < \beta$ (D) $\alpha < 0 < \beta < |\alpha|$

Q.4 a + b + c = 0, then-
(A) Nothing can be said
(B) Unity is one of the roots of the equation
(C) Sum of roots is +ve whereas product is –ve
(D) '0' is one of the roots of the equation

PASSAGE II (Q.NO. 5 - 8) :

If $x \in R$, then the roots of the equation $x^4 + 4x^3 - 8x^2 + k = 0$ when

Q.5 The value of $k \in [0, 3]$, are-
(A) All real
(B) Two real and two complex
(C) No real root
(D) None of these

Q.6 The values of $k \in (3, 4]$, are-
(A) All real
(B) Two real and two complex
(C) No real
(D) Coincident

Q.7 The values of $k \in (-\infty, 0)$ are-
(A) All real
(B) Two real and two imaginary
(C) No real
(D) Repeated roots

Q.8 The values of $k \in (4, \infty)$ are-
(A) All real
(B) Two real and two imaginary
(C) No real
(D) Repeated roots

Q.9 Least natural number 'a' for which $x + ax^{-2} > 2$ for all $x \in (0, \infty)$ is-
(A) 1 (B) 2
(C) 5 (D) None of these

Q.10 All integral values of x, so that $x^2 + 19x + 89$ is a perfect square-
(A) –8, –11 (B) 8, 11
(C) –8, +11 (D) 8, – 11

MATHEMATICS IIT JEE (JULY 5th WEEK CLASS TEST 2) (QUADRATIC EQUATION) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	5	○	○	○	○	9	○	○	○	○
2	○	○	○	○	6	○	○	○	○	10	○	○	○	○
3	○	○	○	○	7	○	○	○	○					
4	○	○	○	○	8	○	○	○	○					

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	B	A	D	B	A	B	B	C	B	A

SOLUTIONS

Sol.1 (B)

Since, ac < 0 and $\alpha < \beta$ where $\alpha.\beta = \frac{c}{a} < 0$

$\Rightarrow$ one root is positive and other root is negative.

Sol.2 (A)

As, a > 0, b > 0, c > 0

$\Rightarrow \quad \alpha + \beta = -\frac{b}{a} < 0$ and $\alpha\beta = \frac{c}{a} > 0$.

$\therefore$ Both have same sign and are negative.

Sol.3 (D)

As, sign of a = sign of b $\neq$ sign of c

$\therefore \quad \alpha + \beta = -\frac{b}{a} < 0$ and $\alpha\beta = \frac{c}{a} < 0$.

$\therefore$ Roots are of opposite sign and the negative one has greater magnitude than positive root.

$\therefore \quad \alpha < 0 < \beta < |\alpha|$.

Sol.4 (B)

As, a + b + c = 0

$\Rightarrow f(x) = ax^2 + bx + c = 0$ has one of the as unit,

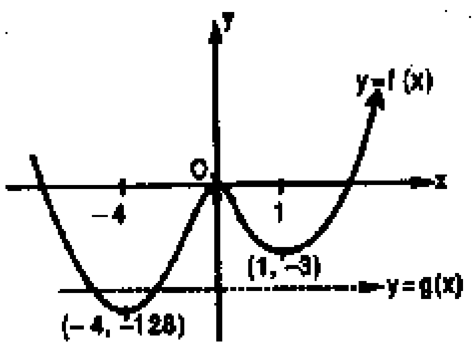
i.e., f(1) = 0

Solution for (5 - 8) :

We have, $x^4 + 4x^3 - 8x^2 + k = 0$

$\Rightarrow \quad x^4 + 4x^3 - 8x^2 = -k$,

Where $f(x) = x^4 + 4x^3 - 8x^2 = x^2(x^2 + 4x - 8)$ could be sketched as,



let, $f(x) = x^4 + 4x^3 - 8x^2$ and $g(x) = -k$

from the graph the following cases arises :

Sol.5 (A)

When $-3 \leq -k \leq 0$, i.e. $0 \leq k \leq 3$

In this case, $y = x^4 + 4x^3 - 8x^2$ and $y = -k$ intersect at four points, so

$x^4 + 4x^3 - 8x^2 + k = 0$ has all real roots.

Sol.6 (B)

When $-4 \leq -k < -3$, i.e. $3 < k \leq 4$

In this case, $y = x^4 + 4x^3 - 8x^2$ and $y = -k$ intersect at two points, to the given equation has two real roots.

Sol.7 (B)

In this case, there are two points of intersection.

So, the equation has two real roots.

Sol.8 (C)

When $-k < -4$, i.e., $k > 4$

In this case, two curves do not intersect, so there is no real root.

Sol.9 ()

Let, $f(x) = x + ax^{-2}$

$\Rightarrow \quad f'(x) = 1 - 2ax^{-3} = 0 \Rightarrow x = (2a)^{1/3}$

$f''(x) = 6ax^{-4} > 0, \ \forall \ x \in (0, \infty)$

(as, 'a' is a natural number)

So, $x = (2a)^{1/3}$ is a point of global minima.

Thus, $(2a)^{1/3} + a(2a)^{-2/3} > 2$

$\Rightarrow \quad a > \frac{32}{27} \Rightarrow a \geq 2$,

as 'a' is the natural number

Sol.10 (A)

Let $x^2 + 19x + 89 = \lambda^2$

$\Rightarrow x^2 + 19x + (89 - \lambda^2) = 0$ should have integral roots

$\therefore \quad$ D should be a perfect square

$\Rightarrow \quad (19)^2 - 4(89 - \lambda^2)$ = perfect square

$\Rightarrow \quad 5 + \lambda^2 4 = m^2$

$\Rightarrow \quad (m^2 - 4\lambda^2) = 5$

$\Rightarrow \quad (m - 2\lambda)(m + 2\lambda) = 5$

$\therefore \quad (m - 2\lambda = 5, m + 2\lambda = 1)$

or $\quad (m - 2\lambda = -5, m + 2\lambda = -1)$

$\Rightarrow \quad m = 3, -3, \lambda = 1, -1$

For $\lambda = \pm 1$, equation becomes

$x^2 + 19x + 88 = 0$

$\Rightarrow \quad (x + 11)(x + 8) = 0$

$\Rightarrow \quad x = -11, -8$.