Dear student following is an Easy level [○ ● ○ ○ ○] test paper. Score of 18 Marks in 10 Minutes would be a satisfactory performance. Questions 1-8(+3,–1). (Questions may have more than one option correct).

Q.1 The total number of solutions of sin {x} = cos {x} [where {.} denotes fractional part of x] in $[0, 2\pi]$ is-

(A) 3 (B) 7 (C) 14 (D) None

Q.2 If $x \in \left[-\frac{5\pi}{2}, \frac{5\pi}{2}\right]$. The greatest +ve solution of $1 + \sin^4 x = \cos^2 3x$ is-

(A) 2π (B) π (C) 3π (D) None

Q.3 If $\sin^2 x + \cos^2 y = 2\sec^2 z$, then-

(A) $x = (2m + 1)\frac{\pi}{2}$ (B) $y = n\pi$

(C) $z = \frac{t\pi}{2}$ (D) None of these

Statement : Let S_1 be the set of all those solutions of the equation $(1 + a)\cos\theta\cos(2\theta - b) = (1 + a\cos 2\theta)\cos(\theta - b)$ which are independent of a and b and S_2 be the set of all such solutions which are dependent on a and b. Then :

Q.4 The set S_1 and S_2 are :

(A) $\{n\pi, n \in Z\}$ and $\{n\pi + (-1)^n \sin^{-1}(a \sin b); n \in Z\}$

(B) $\left\{n\frac{\pi}{2}; n \in Z\right\}$ and $\{n\pi + (-1)^n \sin^{-1}(a \sin b); n \in Z\}$

(C) $\left\{n\frac{\pi}{2}; n \in Z\right\}$ and $\left\{n\pi + (-1)^n \sin^{-1}\left(\frac{a}{2}\sin b\right); n \in Z\right\}$

(D) None of these

Q.5 Conditions that should be imposed on a and b such that S_2 is non-empty :

(A) $\left|\frac{a}{2}\sin b\right| < 1$ (B) $\left|\frac{a}{2}\sin b\right| \le 1$

(C) $|a \sin b| \le 1$ (D) None of these

Q.6 All the permissible values of b, if a = 0 and S_2 is subset of $(0, \pi)$:

(A) $b \in (-n\pi, 2n\pi)$, $n \in Z$

(B) $b \in (-n\pi, 2\pi - n\pi)$, $n \in Z$

(C) $b \in (-n\pi, n\pi)$

(D) None of these

Statement : Equation of form :

$R(\sin x + \cos x, \sin x . \cos x) = 0$

Where R is a rational function of the arguments in the brackets.

Put $\sin x + \cos x = t$ .... (i)

and use the identity

$(\sin x + \cos x)^2 = 1 + 2\sin x\cos x$

$\Rightarrow \sin x \cos x = \frac{t^2 - 1}{2}$ .... (ii)

$\therefore$ Reduce the given equation into,

$R\left(t, \frac{t^2 - 1}{2}\right) = 0$

Q.7 If $(\sin x + \cos x) - 2\sqrt{2}\sin x\cos x = 0$, then x is-

(A) $\left\{2n\pi + \frac{\pi}{4}\right\}$ (B) $\left\{2n\pi - \frac{\pi}{4}\right\}$

(C) $\left\{n\pi + \frac{\pi}{4}\right\}$ (D) $\left\{n\pi - \frac{\pi}{4}\right\}$

Q.8 If $\sin^{10} x + \cos^{10} x = \frac{29}{16} . \cos^4 2x$, then x is-

(A) $\left\{\frac{n\pi}{4}, n \in Z\right\}$ (B) $\{n\pi + \pi, n \in Z\}$

(C) $\left\{\frac{n\pi}{4} + \frac{\pi}{8}\right\}$ (D) None of these

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MATHEMATICS IIT JEE (JULY 1st WEEK CLASS TEST 2) (TRIGONOMETERY) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○					

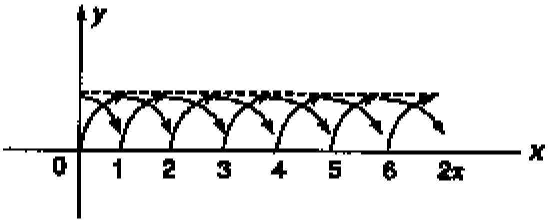
ANSWER KEY

Que.	1	2	3	4	5	6	7	8
Ans.	B	A	A,B	A	C	B	A	C

SOLUTIONS

Sol.1 (B)

Since, $\sin\{X\} = \cos\{x\}$ graphs of $y = \sin\{x\}$ and $y = \cos\{x\}$ meet exactly at 7 points between $[0, 2\pi]$



Sol.2 (A)

$\sin^2 3x + \sin^4 x = 0$

or $\sin^2 x\{(3 - 4\sin^2 x)^2 + \sin^2 x\} = 0$

$\Rightarrow \sin x = 0 \Rightarrow x = n\pi$

Thus, greatest +ve solution is 2π

Sol.3 (A, B)

$\sin^2 x + \cos^2 y = 2\sec^2 z$, only when

$\sin^2 x = 1, \cos^2 y = 1, \sec^2 z = 1$

$\Rightarrow \cos^2 x = 0, \sin^2 y = 0, \cos^2 z = 1$

$\Rightarrow x = (2m + 1)\pi/2, y = n\pi, z = t\pi$

Sol.4-6

4- (A), 5 - (C), 6 - (B)

We have,

$(1 + a)\cos\theta\cos(2\theta - b) = (1 + a\cos 2\theta)\cos(\theta - b)$

$\Rightarrow \cos\theta\cos(2\theta - b) + a\cos\theta\cos(2\theta - b) = \cos(\theta - b) + a\cos 2\theta.\cos(\theta - b)$

$\Rightarrow 2\cos\theta\cos(2\theta - b) + 2a\cos\theta\cos(2\theta - b) = 2\cos(\theta - b) + 2a\cos 2\theta.\cos(\theta - b)$

$\Rightarrow \cos(3\theta - b) + \cos(\theta - b) + a\{\cos(3\theta - b) + \cos(\theta - b)\}$

$= 2\cos(\theta - b) + a\{\cos(3\theta - b) + \cos(\theta + b)\}$

$\Rightarrow \cos(3\theta - b) + a\cos(\theta - b)$

$= \cos(\theta - b) + a\cos(\theta + b)$

$\Rightarrow \cos(3\theta - b) - \cos(\theta - b)$

$= \{\cos(\theta + b) - \cos(\theta + b)\}$

$\Rightarrow 2\sin(2\theta - b)\sin\theta = 2a\sin\theta.\sin b$

$\Rightarrow \sin\theta = 0$ or $\sin(2\theta - b) = a\sin b$

Now $\sin\theta = 0 \Rightarrow \theta = n\pi, n \in Z$

and $\sin(2\theta - b) = a\sin b$

$\Rightarrow \sin(2\theta - b) = a\sin b$

$\Rightarrow 2\theta - b$

$= n\pi + (-1)^n.\sin^{-1}(a\sin b), n \in Z$

(i) Clearly, $S_1 = \{n\pi \mid n \in Z\}$

and $S_2 = \{n\pi + (-1)^n\sin^{-1}(a\sin b); n \in Z\}$

We observe that the equation.

$\sin(2\theta - b) = a\sin b$

is meaningful if, $|a\sin b| \le 1$

(iii) If a = 0, then

$\sin(2\theta - b) = a\sin b$

$\Rightarrow \sin(2\theta - b) = 0$

$\Rightarrow 2\theta - b = n\pi, n \in Z$

$\Rightarrow \theta = \frac{n\pi + b}{2}, n \in Z$

$\Rightarrow S_2 = \left\{\frac{n\pi + b}{2}; n \in Z\right\}$

But it is given that S_2 must be subset of $(0, \pi)$

$\therefore 0 < \frac{n\pi + b}{2} < \pi, n \in Z$

$\Rightarrow 0 < n\pi + b < 2\pi$

$\Rightarrow -n\pi < b < -n\pi + 2\pi$

$\Rightarrow b \in (-n\pi, 2\pi - n\pi), n \in Z$

Sol.7 (A)

Let $(\sin x + \cos x) = t$ and using the equation

$\sin x.\cos x = \frac{t^2 - 1}{2}$, we get

$t - 2\sqrt{2}\left(\frac{t^2 - 1}{2}\right) = 0$

$\Rightarrow \sqrt{2}t^2 - t - \sqrt{2} = 0$

$\therefore t = \sqrt{2}, -\frac{1}{\sqrt{2}}$

$\Rightarrow$ $\sin x + \cos x = \sqrt{2}$ and

$\sin x + \cos x = -\dfrac{1}{\sqrt{2}}$

$\Rightarrow$ $\sin\left(x+\dfrac{\pi}{4}\right) = 1$ and $\sin\left(x+\dfrac{\pi}{4}\right) = \dfrac{-1}{2}$

$\therefore$ $x + \dfrac{\pi}{4} = \dfrac{(4n+1)\pi}{2}$

and $x + \dfrac{\pi}{4} = n\pi + (-1)^n\left(\dfrac{\pi}{6}\right)$

$\Rightarrow$ $x = 2n\pi + \dfrac{\pi}{4}$

and $x = n\pi + (-1)^n\, \pi/6 - \pi/4$

Sol.8 **(C)**

Using the half-angle formula, we get

$$\left(\frac{1-\cos 2x}{2}\right)^5 + \left(\frac{1+\cos 2x}{2}\right)^5$$

$$= \frac{29}{16}\cos^4 2x$$

Put, $\cos 2x = t$

$\therefore$ $\left(\dfrac{1-t}{2}\right)^5 + \left(\dfrac{1+t}{2}\right)^5 = \dfrac{29}{16}t^4$

$\Rightarrow$ $24t^4 - 10t^2 - 1 = 0$ where, $t^2 = 1/2$

$\Rightarrow$ $\cos^2 2x = 1/2 \Rightarrow \cos^4 x = 0$

or $x = \dfrac{n\pi}{4} + \dfrac{\pi}{8}$.