Dear student following is an Easy level [○ ● ○ ○ ○] test paper. Score of 24 Marks in 15 Minutes would be a satisfactory performance. Questions 1-10(+3,–1). (All questions have only one option correct).

Q.1 Sum of the roots of the equation $\tan^2 33x = \cos 2x - 1$ lying in the interval $[\pi, 314]$ is-

(A) 9900π (B) 4950π
(C) 5050π (D) 5049π

Q.2 The number of solutions of the equation $\sin 5x \cos 3x = \sin 6x \cos 2x$ is the interval $[0, 2\pi]$ is-

(A) 1 (B) 5 (C) 3 (D) 4

Q.3 The number of values of x in $[0, 5\pi]$ satisfying the equation $3\cos 2x - 10\cos x + 7 = 0$ is -

(A) 5 (B) 6 (C) 8 (D) 10

Q.4 The sum of all solutions of the equation

$$\cos x . \cos\left(\frac{\pi}{3}+x\right)\cos\left(\frac{\pi}{3}-x\right) = \frac{1}{4}$$

$x \in [0, 6\pi]$ is-

(A) 15π (B) 30π
(C) $\frac{110\pi}{3}$ (D) None of these

Q.5 The general solution of the equation

$$\frac{\sin x}{\cos\frac{3x}{2}\cos\frac{5x}{2}} = 0 \text{ is-}$$

(A) $2m\pi$, $m \in I$ (B) $\frac{8n\pi}{5} + \frac{2\pi}{5}$, $n \in I$
(C) $\frac{(4n+1)}{5}\pi$, $n \in I$ (D) $2(4k+1)\pi$, $k \in I$

Q.6 If $\exp[\{\sin^2 x + \sin^4 x + \sin^6 x + \ldots\ldots \text{ upto } \infty\}\log_e 2]$ satisfies the equation $x^2 - 9x + 8 = 0$ then value of $\frac{\cos x}{\cos x + \sin x}$, $0 < x < \frac{\pi}{2}$ is-

(A) $\frac{\sqrt{3}+1}{4}$ (B) $\frac{\sqrt{3}-1}{2}$
(C) $\frac{\sqrt{3}+2}{4}$ (D) None of these

Q.7 If $1 + \sin\theta + \sin^2\theta + \ldots\ldots \text{ to } \infty = 4 + 2\sqrt{3}$, $0 < \theta < \pi$, $\theta \neq \pi/2$, then

(A) $\theta = \frac{\pi}{6}$ (B) $\theta = \frac{\pi}{3}$
(C) $\theta = \frac{\pi}{3}$ or $\frac{\pi}{6}$ (D) $\theta = \frac{\pi}{3}$ or $\frac{2\pi}{3}$

Q.8 The number of values of $\theta \in [0, 4\pi]$ satisfying the equation $|\sqrt{3}\cos x - \sin x| \geq 2$ is-

(A) 0 (B) 2 (C) 4 (D) 8

Q.9 If $\sin^6\theta = 1 + \cos^4 3\theta$ then the most general value of θ is-

(A) $\left(n+\frac{1}{2}\right)\pi$ (B) $(2n+1)\frac{\pi}{6}$
(C) $\left(n+\frac{1}{2}\right)\frac{\pi}{2}$ (D) None of these

Q.10 If α is a root of $25\cos^2\theta + 5\cos\theta - 12 = 0$, $\frac{\pi}{2} < \alpha < \pi$, then $\sin 2\alpha$ is equal to-

(A) $\frac{24}{25}$ (B) $-\frac{24}{25}$ (C) $\frac{13}{18}$ (D) $-\frac{13}{18}$

MATHEMATICS IIT JEE (JULY 1st WEEK CLASS TEST 1) (TRIGONOMETERY) ANSWER KEY

Name : .... **Roll No. :** ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○
										10	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	B	B	C	B	A	B	D	C	A	B

SOLUTIONS

Sol.1 (B)

$\tan^2 33x = \cos 2x - 1 = 1 - 2\sin^2 x - 1$

$\Rightarrow \tan^2 33x + 2\sin^2 x = 0$

$\Rightarrow \tan 33x = 0, \sin x = 0$

$\Rightarrow x = \frac{n\pi}{33}$, n = 0, 1, 2, ...

or $x = m\pi$, m = 0, 1, 2, ...

in [0, 314] the roots are $x = 0, \pi, 2\pi$, ... 99π $\{\therefore 314 < 100\pi\}$

$\therefore$ Sum $= \pi[1 + 2 + ... + 99]$

$= 99 \times \frac{100}{2}\pi = 4950\pi$.

Sol.2 (B)

Given sin 5x cos 3x = sin 6x cos 2x

$\Rightarrow 2\sin 5x \cos 3x = 2\sin 6x \cos 2x$

$\Rightarrow \sin 8x + \sin 2x = \sin 8x + \sin 4x$

$\Rightarrow \sin 4x - \sin 2x = 0$

$\Rightarrow 2\cos 3x \sin x = 0$

$\Rightarrow \sin x = 0$ and $\cos 3x = 0$

$\Rightarrow x = 0, \pi$ and $3x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$

$\Rightarrow X = 0, \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \pi$

$\Rightarrow$ 5 values.

Sol.3 (C)

$3\cos 2x - 10\cos x + 7 = 0$

$\Rightarrow 3(2\cos^2 x - 1) - 10\cos x + 7 = 0$

$\Rightarrow 6\cos^2 x - 10\cos x + 4 = 0$

$\Rightarrow 3\cos^2 x - 3\cos x - 2\cos x + 2 = 0$

$(\cos x - 1)(3\cos x - 2) = 0$

$\Rightarrow \cos x = 1 \Rightarrow x = 2n\pi$

and $\cos x = \frac{2}{3} \Rightarrow x = 2n\pi \pm \cos^{-1}\left(\frac{2}{3}\right)$

In $[0, 5\pi]$, $x = 2n\pi$ will give one solution for each n = 0, 1, 2 and $x = 2n\pi \pm \cos^{-1}\left(\frac{2}{3}\right)$ will give the solution as follows, for n = 0 one solution, for n = 1 two solution and for n = 2 again two solutions.

Thus total number of solutions is 8.

Sol.4 (B)

We have given

$$\cos x \cos\left(\frac{\pi}{3} + x\right)\cos\left(\frac{\pi}{3} - x\right) = \frac{1}{4}$$

$$\Rightarrow \cos x\left[\cos^2\frac{\pi}{3} - \sin^2 x\right] = \frac{1}{4}$$

$$\Rightarrow \cos x\left(\frac{1}{4} - \sin^2 x\right) = \frac{1}{4}$$

$\Rightarrow \cos x[1 - 4(1 - \cos^2 x)] = 1$

$\Rightarrow 4\cos^3 x - 3\cos x = 1$

or $\cos 3x = 1 = \cos 0$

$\Rightarrow 3x = 2n\pi$

or $x = \frac{2n\pi}{3}$

For $x \in [0, 6\pi]$ we have $0 \le x = \frac{2n\pi}{3} \le 6\pi$

$\Rightarrow 0 \le 2n\pi \le 18\pi$ or $0 \le n \le 9$

Putting n = 0, 1, 2, ... 9 we get ten solutions and their sum is

$$S = 0 + \frac{2\pi}{3} + \frac{4\pi}{3} + ... + \frac{18\pi}{3}$$

$$= \frac{2\pi}{3}[1 + 2 + 3 + ... + 9]$$

$$= \frac{2\pi}{3} \times \frac{9}{2}(9 + 1) = 30\pi.$$

Sol.5 (A)

$$\frac{\sin x}{\cos 3x/2 \cos 5x/2} = 0$$

$\Rightarrow \sin x = 0$ and $\cos\frac{3x}{2} \neq 0$ $\cos\frac{5x}{2} \neq 0$

$\sin x = 0 \Rightarrow x = n\pi$ n = 0, 1, 2, 3, ...

But for n = 1, 3, 5

$\cos\frac{3x}{2} = 0$ $\cos\frac{5x}{2} = 0$

Thus we exclude these odd values of n.

Taking n = 2m we get $x = 2m\pi$.

Sol.6 (B)

We have exp. $[(\sin^2 x + \sin^4 x + \ldots \infty) \log_e 2]$

$$= \text{exp.}\left[\frac{\sin^2 x}{1-\sin^2 x}\log_e 2\right]$$

$$= \text{exp. } [\tan^2 x \log_e 2] = \text{exp. } [\log_e 2^{\tan^2 x}]$$

$$= 2^{\tan^2 x} \quad [\because \text{exp. } [\log_e z] = e^{\log_e z} = z]$$

This satisfies the equation

$x^2 - 9x + 8 = 0 \Rightarrow x = 8, 1$

Thus $2^{\tan^2 x} = 8$ or $2^{\tan^2 x} = 1$

or $2^{\tan^2 x} = 2^3$ or $2^{\tan^2 x} = 2^0$

$\Rightarrow \tan^2 x = 3 \qquad \Rightarrow \tan x = \sqrt{3}$

$\Rightarrow x = \frac{\pi}{3}$

or $\tan^2 x = 0$

But given that $0 < x < \frac{\pi}{2} \Rightarrow x = \frac{\pi}{3}$

Now $\frac{\cos x}{\cos x + \sin x} = \frac{\cos \pi/3}{\cos \pi/3 + \sin \pi/3}$

$$= \frac{1/.2}{1/2 + \sqrt{3}/2} = \frac{1}{1+\sqrt{3}} = \frac{\sqrt{3}-1}{2}$$

Sol.7 (D)

The left hand side of the given equation is an infinite G.P. with common ratio $\sin\theta$. Since $0 < \theta < \pi$, $\theta \neq \pi/2$, we have $0 < \sin\theta < 1$. We now sum up the infinite G.P. so that the given equation becomes

$1/(1 - \sin\theta) = 4 + 2\sqrt{3}$

or $1 - \sin\theta = \frac{1}{(4+2\sqrt{3})} = \frac{4-2\sqrt{3}}{16-12}$

$= 1 - \frac{\sqrt{3}}{2} \quad \therefore \quad \sin\theta = \frac{\sqrt{3}}{2} = \sin\frac{\pi}{3}$

$\therefore \quad \theta = \frac{\pi}{3}$ or $\pi - \frac{\pi}{3} = \frac{2\pi}{3}$

in the interval $0 < \theta < \pi$.

Sol.8 (C)

L.H.S. $= \left|2\cos\left(x + \frac{\pi}{6}\right)\right|$

Also max. value of $2\cos\left(x + \frac{\pi}{6}\right)$ is 2. Hence we must have the sign of equality i.e.

$\left|2\cos\left(x + \frac{\pi}{6}\right)\right| = 2$

ir $\left|\cos\left(x + \frac{\pi}{6}\right)\right| = 1$ i.e. $\cos\left(x + \frac{\pi}{6}\right) = 1, -1$

$x + \frac{\pi}{6} = 0, 2\pi, 4\pi$ and $\pi, 3\pi$

$\therefore \quad x = -\frac{\pi}{6}, \frac{11\pi}{6}, \frac{23\pi}{6}, \frac{5\pi}{5}, \frac{17\pi}{6}$.

Rejecting $-\frac{\pi}{6}$ as it does not belong to $[0, 4\pi]$

Sol.9 (A)

L.H.S. ≤ 1 where R.H.S. > 1 always.

For equality of both sides we must have each side equal to 1.

$\therefore \quad \sin^6\theta = 1$ and $1 + \cos^4 3\theta = 1$

or $\sin^2\theta = 1 \qquad \cos^4 3\theta = 0$

$\therefore \quad \cos\theta = 0 \qquad \cos 3\theta = 0$

$\therefore \quad \theta = \frac{\pi}{2}, \frac{3\pi}{2} \qquad 3\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

or $\theta = \frac{\pi}{2}, \frac{3\pi}{2} \qquad \theta = \frac{\pi}{6}, \frac{\pi}{2}$

The common value in $(0, 2\pi)$ is $\frac{\pi}{2}$

Hence the general value in $\left(2n\pi + \frac{\pi}{2}\right)$

$= (4n + 1)\frac{\pi}{2} = \text{odd}\frac{\pi}{2} = (2n + 1)\frac{\pi}{2}$

$= \left(n + \frac{1}{2}\right)\pi$

Sol.10 (B)

$\cos\alpha = \frac{-5 \pm 35}{50} = -\frac{4}{5}$ or $\frac{3}{5}$

But $\frac{\pi}{2} < \alpha < \pi$

$\Rightarrow \cos\alpha$ is –ve, $\sin\alpha$ = + ive.

We choose $\cos\alpha = -4/5$

$\Rightarrow \sin\alpha = 3/5$ (+ive)

$\therefore \quad \sin 2\alpha = 2\sin\alpha\cos\alpha$

$= 2 \cdot \left(-\frac{4}{5}\right)\left(\frac{3}{5}\right) = -\frac{24}{25}$