Dear student following is a Moderate level [○○●○○] test paper. Score of 15 Marks in 10 Minutes would be a satisfactory performance. Questions 1-10(+3,–1). (Questions may have more than one option correct)

Q.1 If sin x + sin y = sin (x + y) and |x| + |y| = 1, the no. of solutions are

(A) 1 (B) 2 (C) 4 (D) 3

Q.2 $\frac{\sec^2\theta - \tan\theta}{\sec^2\theta + \tan\theta}$ lies between

(A) $\left[\frac{1}{3}, 3\right]$ (B) $\left(\frac{1}{3}, 3\right)$

(C) $\left[\frac{1}{3}, 3\right)$ (D) $\left(\frac{1}{3}, 3\right]$

Q.3 ($\tan\alpha$ + 2 tan 2α + 4 tan 4α + ... + n terms) – cot α + 2^n cot 2^n α =

(A) –1 (B) 2 (C) 0 (D) –2

Q.4 If $\pi < \theta < \frac{3\pi}{2}$ the expression

$\sqrt{4\sin^4\theta + \sin^2 2\theta}$ + 4 $\cos^2\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$ is equal to

(A) 2 (B) 2 + 4 $\sin\theta$

(C) 2 – 4 $\sin\theta$ (D) None of these

Q.5 If $\frac{3\pi}{4} < \alpha < \pi$, then $\sqrt{2\cot\alpha + \frac{1}{\sin^2\alpha}}$ is equal to

(A) 1 + $\cot\alpha$ (B) – 1 – $\cot\alpha$

(C) 1 – $\cot\alpha$ (D) – 1 + $\cot\alpha$

Q.6 If sin A = sin B and cos A = cos B, then

(A) sin (1/2) (A – B) = 0

(B) sin (1/2) (A + B) = 0

(C) cos (1/2) (A – B) = 0

(D) cos (1/2) (A + B) = 0

Q.7 The value of tan 3α $\cot\alpha$ cannot lie in

(A)]0, 2/3[(B)]1/3, 3[

(C)]4/3, 4[(D)]2, 10/3[

Q.8 If $x_{n+1} = \sqrt{\frac{1}{2}(1 + x_n)}$, then

$\cos\left[\frac{\sqrt{1 - x_0^2}}{x_1 x_2 x_3 \ldots \text{to infinite}}\right]$ $(-1 \le x_0 \le 1)$ is equal to

(A) –1 (B) 1 (C) x_0 (D) $1/x_0$

Q.9 For 0 < θ < $\pi/2$, tan θ + tan 2θ + tan 3θ = 0 if

(A) tan θ = 0 (B) tan 2θ = 0

(C) tan 3θ = 0 (D) tan θ tan 2θ = 2

Q.10 If A and B are acute angle such that sinA = $\sin^2$ B, 2 $\cos^2$ A = 3 $\cos^2$ B, then

(A) A = $\pi/6$ (B) A = $\pi/2$

(C) B = $\pi/4$ (D) B = $\pi/3$

MATHEMATICS IIT JEE (JUNE 4Th WEEK CLASS TEST 2) (TRIGONOMETERY) ANSWER

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○
										10	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	C	A	C	A	B	A	B	C	C,D	A,C

SOLUTIONS

Sol.1 (C)

$2\sin\frac{x+y}{2}\cos\frac{x-y}{2} = 2\sin\frac{x+y}{2}\cos\frac{x+y}{2}$

$\Rightarrow \cos\frac{x+y}{2} = \cos\frac{x-y}{2}$

$\Rightarrow x = 0$ or $y = 0$

Because $|x| \le 1, |y| \le 1$

$|x| + |y| = 1,$

for $x = 0, y = \pm 1$

for $y = 0, x = \pm 1$

Sol.2 (A)

Let $y = \frac{\sec^2\theta - \tan\theta}{\sec^2\theta + \tan\theta}$

$y = \frac{\tan^2\theta - \tan\theta + 1}{\tan^2\theta + \tan\theta + 1} = \frac{x^2 - x + 1}{x^2 + x + 1}$

where $x = \tan\theta \in R$

Hence $(y - 1)x^2 + (y + 1)x + (y - 1) = 0$

has real roots

$\therefore (y + 1)^2 - (2y - 2)^2 \ge 0$

Gives $(3y - 1)(3 - y) \ge 0$

$\therefore \frac{1}{3} \le y \le 3$

Sol.3 (C)

Given

$(-\cot\alpha + \tan\alpha) + 2\tan 2\alpha + 4\tan 4\alpha + \ldots + 2^n\cot 2^n\alpha$

$= \left(-\frac{\cos\alpha}{\sin\alpha} + \frac{\sin\alpha}{\cos\alpha}\right) + 2\tan 2\alpha + 4\tan 4\alpha + \ldots + 2^n\cot 2^n\alpha$

$= -\frac{2(\cos^2\alpha - \sin^2\alpha)}{2\sin\alpha\cos\alpha} + 2\tan 2\alpha + 4\tan 4\alpha + \ldots + 2^n\cot 2^n\alpha$

$= (-2\cot 2\alpha + 2\tan 2\alpha) + 4\tan 4\alpha + \ldots + 2^n\cot 2^n\alpha$

$= -4\cot 4\alpha + 4\tan 4\alpha + \ldots + 2^n\cot 2^n\alpha$

$= (-2^2\cot 2^2\alpha + 2^2\tan 2^2\alpha) + \ldots + 2^n\cot 2^n\alpha$

Proceeding as above, we get

$= -2^n\cot 2^n\alpha + 2^n\cot 2^n\alpha$

$= 0$

Sol.4 (A)

Given expression

$= \sqrt{4\sin^4\theta + \sin^2 2\theta} + 4\cos^2\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$

$= \sqrt{4\sin^4\theta + 4\sin^2\theta\cos^2\theta} + 2.2\cos^2\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$

$= \sqrt{4\sin^2\theta(\sin^2\theta + \cos^2\theta)} + 2\left[1 + \cos 2\left(\frac{\pi}{4} - \frac{\theta}{2}\right)\right]$

$= 2\sqrt{\sin^2\theta} + 2 + 2\cos\left(\frac{\pi}{2} - \theta\right)$

$= 2|\sin\theta| + 2\sin\theta + 2 \qquad [\because \sqrt{z^2} = |z|]$

$= -2\sin\theta + 2\sin\theta + 2$

$\left[\because \pi < \pi < \frac{3\pi}{2} \therefore \sin\theta \text{ is } (-)\text{ve}\right]$

$= 2$

Sol.5 (B)

$\sqrt{2\cot\alpha + \frac{1}{\sin^2\alpha}} = \sqrt{2\cot\alpha + \text{cosec}^2\alpha}$

$\sqrt{2\cot\alpha + 1 + \cot^2\alpha} = \sqrt{(1 + \cot\alpha)^2} = |1 + \cot\alpha|$

Since $\cot\alpha < -1$ when $\frac{3\pi}{4} < \alpha < \pi$,

we have $|1 + \cot\alpha| = -1 - \cot\alpha$.

Sol.6 (A)

$\sin A = \sin B$

$\Rightarrow \sin A - \sin B = 0$

$\Rightarrow 2\sin\frac{A-B}{2}\cos\frac{A+B}{2} = 0$ ....(i)

and $\cos A = \cos B$

$\Rightarrow \cos B - \cos A = 0$

$\Rightarrow 2\sin\frac{A+B}{2}\sin\frac{A-B}{2} = 0$ ....(ii)

Equation (i) and (ii) are simultaneously true if $\sin\left(\frac{1}{2}\right)(A - B) = 0$, while the other factors, $\sin\left(\frac{1}{2}\right)(A + B)$ and $\cos\left(\frac{1}{2}\right)(A + B)$ cannot both be zero simultaneously.

Sol.7 (B)

$$\tan 3\alpha \cot\alpha = \frac{3\tan\alpha - \tan^3\alpha}{\tan\alpha(1-3\tan^2\alpha)}$$

$$= \frac{3-\tan^2\alpha}{1-3\tan^2\alpha} = x \text{ (say)}$$

$$\Rightarrow \tan^2\alpha = \frac{x-3}{3x-1} = \frac{(3x-1)(x-3)}{(3x-1)^2}$$

Since $\tan^2\alpha$ is nonnegative, either $x < \frac{1}{3}$ or $x \geq 3$, so x cannot lie between $\frac{1}{3}$ and 3.

Sol.8 (C)

Let $x_0 = \cos\theta$, then $x_1 = \sqrt{\frac{1}{2}(1+\cos\theta)} = \cos\frac{\theta}{2}$, $x_2 = \cos\left(\frac{\theta}{2^2}\right)$, $x_3 = \cos\left(\frac{\theta}{2^3}\right)$, ... and so on.

so that $\left[\frac{\sqrt{1-x_0^2}}{x_1x_2x_3..\text{to infinite}}\right]$

$$= \frac{\sin\theta}{\cos\frac{\theta}{2}\cos\frac{\theta}{2^2}....\cos\frac{\theta}{2^n}.....\text{infinite}}$$

$$= \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\cos\frac{\theta}{2}\cos\frac{\theta}{2^2}....\cos\frac{\theta}{2^n}.....\text{infinite}}$$

$$= \frac{2^2\sin\frac{\theta}{2^2}\cos\frac{\theta}{2^2}}{\cos\frac{\theta}{2^2}\cos\frac{\theta}{2^3}....\cos\frac{\theta}{2^n}.....\text{infinite}}$$

$$= \lim_{n\to\infty}\frac{2^n\sin\frac{\theta}{2^n}}{\cos\frac{\theta}{2^{n+1}}}$$

$$= \lim_{n\to\infty}\theta\left(\frac{\sin\frac{\theta}{2^n}}{\frac{\theta}{2^n}}\right)\frac{1}{\cos\frac{\theta}{2^{n+1}}} = \theta$$

so that $\cos\left[\frac{\sqrt{1-x_0^2}}{x_1x_2..\text{ inf.}}\right] = \cos\theta = x_0$.

Sol.9 (C, D)

Clearly $\tan\theta \neq 0$ and $\tan 2\theta \neq 0$

for $0 < \theta < \frac{\pi}{2}$. We have

$\tan 3\theta = \tan(2\theta + \theta)$

$\Rightarrow \tan 3\theta = \frac{\tan 2\theta + \tan\theta}{1-\tan 2\theta\tan\theta}$

Given $0 = \tan\theta + \tan 2\theta + \tan 3\theta$

$= \tan 3\theta\,(1 - \tan 2\theta\,\tan\theta) + \tan 3\theta$

$= \tan 3\theta\,[2 - \tan 2\theta\,\tan\theta]$

$\Rightarrow \tan 3\theta = 0$

or $\tan 2\theta\,\tan\theta = 2$.

Sol.10 (A, C)

From the given conditions

$2(1 - \sin^2 A) = 3(1 - \sin^2 B) = 3(1 - \sin A)$

$\Rightarrow 2\sin^2 A - 3\sin A + 1 = 0$

$\Rightarrow (2\sin A - 1)(\sin A - 1) = 0$

$\Rightarrow \sin A = 1$ or $\sin A = 1/2$

$\Rightarrow A = \frac{\pi}{2}$ or $\frac{\pi}{6}$

But since A is acute, we have $A = \frac{\pi}{6}$

$\Rightarrow \sin^2 B = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

$\Rightarrow \sin B = \frac{1}{\sqrt{2}} \quad \Rightarrow \quad B = \frac{\pi}{4}$