Dear student following is a Moderate level [○○●○○] test paper. Score of 18 Marks in 10 Minutes would be a satisfactory performance. Questions 1-10(+3,–1). (All questions have only one option correct)

Q.1 In a ΔABC, the maximum value of

$\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$ is

(A) $\frac{1}{8}$ (B) $3\frac{\sqrt{3}}{8}$ (C) $\frac{1}{2\sqrt{2}}$ (D) 1

Q.2 Value of $(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2$ is greater than equal to

(A) $\frac{2}{9}$ (B) 9 (C) 7 (D) $\frac{9}{2}$

Q.3 $0 \le a \le 3$, $0 \le b \le 3$ and the equation, $x^2 + 4 + 3\cos(ax + b) = 2x$ has at least one solution then the value of $(a + b)$ =

(A) π (B) 2π (C) $\frac{\pi}{2}$ (D) 3π

Q.4 All possible real values of x and y satisfying $\sin^2x + 4\sin^2y - \sin x - 2\sin y$

$- 2\sin x \sin y + 1 = 0$, $\forall x, y \in \left[0, \frac{\pi}{2}\right]$ is /are-

(A) 1 (B) 2 (C) 3 (D) None

Q.5 All number paris x, y that satisfy the equation $\tan^4x + \tan^4y + 2\cot^2x\cot^2y$

$= 3 + \sin^2(x + y)$, $\forall x, y \in \left[0, \frac{\pi}{2}\right]$ is/are-

(A) 2 (B) No value (C) 1 (D) 3

Q.6 If $\tan\alpha$ equals the integral solutions of the inequality $4x^2 - 16x + 15 < 0$ and $\cos\beta$ equals the slope of the bisectors of the first quadrant, then the value of $\sin(\alpha + \beta)\sin(\alpha - \beta)$ is

(A) 1 (B) $\frac{1}{5}$ (C) $\frac{4}{5}$ (D) 2

Q.7 The minimum value of the expression $\sin\alpha + \sin\beta + \sin\lambda$, when α, β, λ are real numbers satisfying $\alpha + \beta + \lambda = \pi$ is

(A) –3 (B) Negative

(C) Positive (D) Zero

Q.8 $\alpha, \beta, \gamma \in \left(0, \frac{\pi}{2}\right)$, then $\frac{\sin(\alpha+\beta+\gamma)}{\sin\alpha+\sin\beta+\sin\gamma}$ is

(A) < 1 (B) >1

(C) = 1 (D) None of these

The following questions consist of two statements one labelled Assertion (A) and the another labelled Reason (R). Select the correct answers to these questions from the codes given below :

(A) Both A and R are true and R is the correct explanation of A.
(B) Both A and R are true but R is not correct explanation of A
(C) A is true but R is false
(D) A is false but R is true.

Q.9 **Assertion :** If A,B,C,D be the angles of cyclic quadrilateral then

$\cos A + \cos B + \cos C + \cos D = 0$

Reason : $\sin A + \sin B + \sin C + \sin D = 0$

Q.10 **Assertion :** $\sec^2\theta = \frac{4xy}{(x+y)^2} \Rightarrow x = y$

Reason : $\sec^2\theta \ge 1$

MATHEMATICS IIT JEE (JUNE 4Th WEEK CLASS TEST 3) (TRIGONOMETERY) ANSWER

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○
										10	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	A	B	C	C	C	C	C	A	C	A

SOLUTIONS

Sol.1 (A)

As we know in Δ ABC cosA + cosB + cosC =

$1 + 4\sin\frac{A}{2}\ \sin\frac{B}{2}\ \sin\frac{C}{2}$

and since $\cos A + \cos B + \cos C \le \frac{3}{2}$

$\therefore$ we get $\sin\frac{A}{2}\ \sin\frac{B}{2}\ \sin\frac{C}{2} \le \frac{1}{8}$

Sol.2 (B)

$(\sin\theta + \text{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2$

$= \sin^2\theta + \text{cosec}^2\theta + 2 + \cos^2\theta + \sec^2\theta + 2$

$= (\sin^2\theta + \cos^2\theta) + 4 + (1 + \cos^2\theta) + (1 + \tan^2\theta)$

$= 7 + \cos^2\theta + \tan^2\theta$ ....(1)

As we know

A.M $\ge$ G.M.

$\therefore \quad \frac{\cot^2\theta + \tan^2\theta}{2} \ge \sqrt{\tan^2\theta\cot^2\theta}$

$\Rightarrow \tan^2\theta + \cot^2\theta \ge 2$ ...(2)

$\therefore$ From (1) and (2)

$(\sin\theta + \text{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 \ge 7 + 2 = 9$

Sol.3 (C)

$x^2 - 2x + 4 = -\cos(ax + b)$.

$\Rightarrow (x - 1)^2 + 3 = -3\cos(ax + b)$ ... (1)

As $-1 \le \cos(ax + b) \le 1$ and $(x - 1)^2 \ge 0$

$\therefore$ equation (1) is only possible if,

$\cos(ax + b) = -1$ and $(x - 1) = 0$.

So $a + b = \pi, 3\pi, 5\pi, ...$]

and $3\pi > 6$ where $a + b \le 6$

$\Rightarrow a + b = \pi$

Sol.4 (C)

Given equation

$\sin^2x + 4\sin^2y - \sin x - 2\sin y - 2\sin x \sin y + 1 = 0$

$\Rightarrow \sin^2x - \sin x(1 + 2\sin y) + 4\sin^2y - 2\sin y + 1 = 0$

$\Rightarrow \sin x = \frac{(1 + 2\sin y) \pm \sqrt{(1 + 2\sin y)^2 - 4(4\sin^2 y - 2\sin y + 1)}}{2}$

$\sin x = \frac{(1 + 2\sin y) \pm \sqrt{3(2\sin y - 1)^2}}{2}$

Now since sin x is real

$\Rightarrow$ given equation is real if

$2\sin y - 1 = 0$ or $\sin y = \frac{1}{2}$

$\Rightarrow \sin x = \frac{1 + 1}{2} = 1$

$\therefore \quad x = \frac{\pi}{2}$ and $y = \frac{\pi}{6}$ as $x, y \in \left[0, \frac{\pi}{2}\right]$

Sol.5 (C)

We know, $a^4 + b^4 \ge 2a^2b^2$ {A.M $\ge$ G.M. }

$\therefore \tan^4x + \tan^4y \ge 2\tan^2x\tan^2y$ ...(1)

Equality occuring only when $\tan^2x = \tan^2y = 1$.

Also, $\tan^2x.\tan^2y + \cot^2x.\cot^2y \ge 2$ ..(2)

Since, $a + \frac{1}{a} \ge 2$ and equality occuring only when a = 1, i.e., $\tan^2x.\tan^2y = 1$

from (1) and (2);

$\Rightarrow \tan^4x + \tan^4y + 2\cot^2x.\cot^2y \ge 4$ ...(3)

Also, R.H.S. $= 3 + \sin^2(x + y) \le 4$ ...(4)

from (3) and (4)

L.H.S. = R.H.S. = 4

$\Rightarrow \tan^2x = \tan^2y = \tan^2x\tan^2y = 1$

$\Rightarrow \tan x = \tan y = \pm 1$

$\Rightarrow \tan x = \tan y = 1$ {as x, y [0, $\pi/2$]}

$\therefore x = y = \pi/4$

Only one solution i.e., ($x = \pi/4$, $y = \pi/4$).

Sol.6 (C)

We have the inequality $4x^2 - 16x + 15 < 0$ ..(1)

$\Rightarrow (2x - 3)(2x - 5) < 0$

$\Rightarrow \frac{3}{2} < x < \frac{5}{2}$

The only integral solution of the inequality is 2 since $\tan\alpha$ is the integral soultion of (1).

Therefore, $\tan\alpha = 2$ ....(2)

Also slope of the bisector of first quadrant is $\tan 45^0 = 1$

$\therefore \cos\beta = 1$ ....(3)

Now $\sin(\alpha + \beta)\sin(\alpha - \beta)$

$= \sin^2\alpha - \sin^2\beta = \cos^2\beta - \cos^2\alpha$

$= \cos^2\beta - \frac{1}{\sec^2\alpha} = \cos^2\beta - \frac{1}{1 + \tan^2\alpha}$

$= (1)^2 - \frac{1}{1 + (2)^2}$ [From (2) and (3)]

$= 1 - \frac{1}{5} = \frac{4}{5}$

Sol.7 (C)

We have,

$$\sin\alpha + \sin\beta + \sin\lambda = 4\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\lambda}{2}$$

$\because$ each of $\frac{\alpha}{2}, \frac{\beta}{2}, \frac{\lambda}{2}$ is $\frac{\pi}{2}$

$\therefore$ $\cos\frac{\alpha}{2}, \cos\frac{\beta}{2}, \cos\frac{\lambda}{2}$ are all + ve

Hence, mimimum value of $\sin\alpha + \sin\beta + \sin\lambda$ is + ve.

Sol.8 (A)

We have $\sin\alpha + \sin\beta + \sin\gamma - \sin(\alpha + \beta + \gamma)$

$= \sin\alpha + \sin\beta + \sin\gamma - \sin\alpha\cos\beta\cos\gamma - \cos\alpha \sin\beta\cos\gamma - \cos\alpha\cos\beta\sin\gamma + \sin\alpha\sin\beta\sin\gamma$

$= \sin\alpha(1 - \cos\beta\cos\gamma) + \sin\beta(1 - \cos\alpha\cos\gamma) + \sin\gamma(1 - \cos\alpha\cos\beta) + \sin\alpha\sin\beta\sin\gamma > 0$

$\therefore$ $\sin\alpha + \sin\beta + \sin\gamma > \sin(\alpha + \beta + \gamma)$

$$\Rightarrow \frac{\sin(\alpha+\beta+\gamma)}{\sin\alpha+\sin\beta+\sin\gamma} < 1.$$

Sol.9 (C)

Assertion is true but reason is false

In a cyclic quadriletral $A + C = \pi$, $B + D = \pi$

$\therefore$ $\cos C = -\cos A$ and $\cos D = -\cos B$

$\Rightarrow$ $\Sigma\cos A = 0$

Sol.10 (A)

We know $\sec^2\theta \geq 1$ and $\frac{4xy}{(x+y)^2} \leq 1$

{as A.M $\geq$ G.M }

$\Rightarrow$ $\sec^2\theta = \frac{4xy}{(x+y)^2}$ is possible if $\sec^2\theta = 1$

$\Rightarrow$ $\frac{4xy}{(x+y)^2} = 1 \Rightarrow 4xy = (x + y)^2$

$\Rightarrow$ $x^2 + y^2 - 2xy = 0 \Rightarrow (x-y)^2 = 0$

$\Rightarrow$ $x = y \quad \forall\, x, y \in R^+$