Dear student following is a Moderate level [O O ● O O] test paper. Score of 18 Marks in 10 Minutes would be a satisfactory performance. Questions 1-10(+3,–1). (All questions have only one option correct)

Q.1 The expression $\sec^4 x - 4\tan^3 x + 4\tan x$ is always-
(A) +ve (B) –ve
(C) Non negative (D) Non positive

Q.2 Given that $\sin\beta = \frac{12}{13}, 0 < \beta < \pi$, then $\{5\sin(\alpha+\beta) - 12\cos(\alpha+\beta)\}\operatorname{cosec}\alpha$ is equal to-
(A) 13 if $\tan\beta > 0$ (B) 13 if $\tan\beta < 0$
(C) $119 + 120\cos\alpha$, if $\tan\beta < 0$
(D) $119 + 120\cot\alpha$, if $\tan\beta > 0$

Q.3 If k = (1 + sin A) (1 + sin B) (1 + sin C) = (1 – sin A) (1 – sin B) (1 – sin C). Then k =
(A) ±cos A cos B cos C (B) ±sin A sin B sin C
(C) ±tan A tan B tan C (D) ±cot A cot B cot C

Q.4 If $\alpha, \beta, \gamma, \delta$ are the smallest positive angles in ascending order of magnitude which have their sines equal to the positive number x, then the value of
$4\sin\frac{\alpha}{2} + 3\sin\frac{\beta}{2} + 2\sin\frac{\gamma}{2} + \sin\frac{\delta}{2}$ is equal to-
(A) $2\sqrt{1-x}$ (B) $2\sqrt{1+x}$
(C) $2\sqrt{x}$ (D) None of these

Q.5 The set of values of α for which the point $(\sin\alpha, \cos\alpha)$ does not lie outside the curve $2y^2 + x - 2 = 0$ in the interval $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$
(A) $\left(\frac{\pi}{2}, \frac{5\pi}{6}\right)$ (B) $\left(\frac{\pi}{2}, \frac{5\pi}{6}\right]$
(C) $\left(\pi, \frac{5\pi}{6}\right)$ (D) $\left[\frac{\pi}{2}, \frac{5\pi}{6}\right] \cup \left[\pi, \frac{3\pi}{2}\right]$

Q.6 If $\cos\theta = \sqrt{q}$, where q is a rational number and $\sqrt{q}$ is an irrational number, then the integral values of n, for which $\cos n\theta$ is a rational number-
(A) If n is even (B) If n is odd
(C) If neither even nor odd
(D) None of these

Q.7 If $1 + \cot\theta \le \cot\frac{\theta}{2}$ for $0 < \theta < \pi$, then equality holds for θ =
(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) $\frac{3\pi}{2}$ (D) 2π

Q.8 If $\alpha, \beta, \gamma, \delta$ are the solutions of the equation $\tan\left(\theta + \frac{\pi}{4}\right) = 3\tan 30$ no two of which have equal tangents, then value of $\tan\alpha + \tan\beta + \tan\gamma + \tan\delta$ equals to-
(A) 0 (B) – 2 (C) $\frac{8}{3}$ (D) $\frac{1}{3}$

Q.9 Value of $\sin^2 12 + \sin^2 21 + \sin^2 39 + \sin^2 48 - \sin^2 9 - \sin^2 18$ =
(A) –1 (B) 1 (C) $\frac{1}{2}$ (D) None

Q.10 If $\frac{\pi}{2} < \alpha < \pi$, then the expression $\left|\sqrt{\frac{1-\sin\alpha}{1+\sin\alpha}} + \sqrt{\frac{1+\sin\alpha}{1-\sin\alpha}}\right|$ is equal to-
(A) $\frac{2}{\cos\alpha}$ (B) $-\frac{2}{\cos\alpha}$ (C) $\frac{2}{\sin\alpha}$ (D) $\frac{2}{\tan\alpha}$

MATHEMATICS IIT JEE (JUNE 4Th WEEK CLASS TEST 4) (TRIGONOMETERY) ANSWER

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	⬭	⬭	⬭	⬭	4	⬭	⬭	⬭	⬭	7	⬭	⬭	⬭	⬭
2	⬭	⬭	⬭	⬭	5	⬭	⬭	⬭	⬭	8	⬭	⬭	⬭	⬭
3	⬭	⬭	⬭	⬭	6	⬭	⬭	⬭	⬭	9	⬭	⬭	⬭	⬭
										10	⬭	⬭	⬭	⬭

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	D	A	A	B	D	A	A	A	B	B

SOLUTIONS

Sol.1 (D)

$\sec^4 x - 4\tan^3 x + 4\tan x$

$= \sec^4 x - 4\tan^2 x \cdot \tan x + 4\tan x$

$= \sec^4 x - 4(\sec^2 x - 1)\tan x + 4\tan x$

$= \sec^4 x - 4\sec^2 x \tan x + 8\tan x$

$= (\sec^2 x - 2\tan x - 2)^2 - 4\tan^2 x - 4 + 4\sec^2 x$

$= (\sec^2 x - 2\tan x - 2)^2 \geq 0$

$[\because \sec^2 x - \tan^2 x = 1]$

Thus equality holding for

$$x = \tan^{-1}(1 \pm \sqrt{2}) = \frac{3\pi}{8} \text{ or } \frac{7\pi}{8}$$

Sol.2 (A)

$\sin\beta = \frac{12}{13} \Rightarrow \cos\beta = \pm\frac{5}{13}$ according as

$\tan\beta >$ or < 0

$\therefore 5\sin(\alpha+\beta) - 12\cos(\alpha+\beta)$

$= 5[\sin\alpha\cos\beta + \cos\alpha\sin\beta]$

$- 12[\cos\alpha\cos\beta - \sin\alpha\sin\beta]$

$= (5\cos\beta + 12\sin\beta)\sin\alpha$

$+ (5\sin\beta - 12\cos\beta)\cos\alpha$

$$= \left(\frac{25}{13} + \frac{144}{13}\right)\sin\alpha + \left(\frac{60}{13} - \frac{60}{13}\right)\cos\alpha$$

$= 13\sin\alpha$ if $\tan\beta > 0$

$\therefore \{5\sin(\alpha+\beta) - 12\cos(\alpha+\beta)\}$

$\text{cosec}\,\alpha = 13$, if $\tan\beta > 0$

If $\tan\beta < 0$, then

$5\sin(\alpha+\beta) - 12\cos(\alpha+\beta)$

$$= \left(-\frac{25}{13} + \frac{144}{13}\right)\sin\alpha + \left(\frac{60}{13} + \frac{60}{13}\right)\cos\alpha$$

$$= \frac{119}{13}\sin\alpha + \frac{120}{13}\cos\alpha$$

Hence, $[5\sin(\alpha+\beta) - 12\cos(\alpha+\beta)]$

$\text{cosec}\,\alpha = 13$ if $\tan\beta > 0$.

Sol.3 (A)

Since $(1 + \sin A)(1 + \sin B)(1 + \sin C) = (1 - \sin A)(1 - \sin B)(1 - \sin C)$ ...(1)

Multiplying both sides by $(1 - \sin A)(1 - \sin B)(1 - \sin C)$, we get

$(1 - \sin^2 A)(1 - \sin^2 B)(1 - \sin^2 C)$

$= (1 - \sin A)^2(1 - \sin B)^2(1 - \sin C)^2$

$\Rightarrow (1 - \sin A)(1 - \sin B)(1 - \sin A)$

$= \pm\cos A\cos B\cos C$

From (1), R.H.S. $= \pm\cos A\cos B\cos C$

L.H.S. $= \pm\cos A\cos B\cos C$

Sol.4 (B)

If α is the smallest positive angle for which $\sin a = x$, then $\beta = \pi - \alpha$, $\gamma = 2\pi + \alpha$ and $\delta = 3\pi - \alpha$

So, $4\sin\frac{\alpha}{2} + 3\sin\frac{\beta}{2} + 2\sin\frac{\gamma}{2} + \sin\frac{\delta}{2}$

$= 4\sin\frac{\alpha}{2} + 3\cos\frac{\beta}{2} - 2\sin\frac{\gamma}{2} - \cos\frac{\delta}{2}$

$= 2\sin\frac{\alpha}{2} + 2\cos\frac{\alpha}{2}$

$= 2\sqrt{1+\sin\alpha} = 2\sqrt{1+x}$

Sol.5 (D)

Point $(\sin\alpha, \cos\alpha)$ does not lie outside the (parabola) curve $2y^2 + x - 2 = 0$

$\Rightarrow 2\cos^2\alpha + \sin\alpha - 2 \leq 0$

$\Rightarrow 2 - 2\sin^2\alpha + \sin\alpha - 2 \leq 0$

$\Rightarrow \sin\alpha(1 - 2\sin\alpha) \leq 0$

Case I :

$\sin\alpha \geq 0$ and $(1 - 2\sin\alpha) \leq 0$

$\Rightarrow 0 \leq \alpha \leq \pi$ and $\frac{\pi}{6} \leq \alpha \leq \frac{5\pi}{6}$

$\Rightarrow \alpha \in \left[\frac{\pi}{2}, \frac{5\pi}{6}\right]$

Case II :

$\sin\alpha \leq 0$ and $(1 - 2\sin\alpha) \geq 0$

$\Rightarrow \pi \leq \alpha \leq 2\pi$ and $0 \leq \alpha \leq \frac{\pi}{6}$

or $\frac{5\pi}{6} \leq \alpha \leq 2\pi$

$\Rightarrow \alpha \in \left[\frac{\pi}{2}, \frac{5\pi}{6}\right] \cup \left[\pi, \frac{3\pi}{2}\right]$

Sol.6 (A)

As we know $(\cos\theta + i\sin\theta)^n$

$= (\cos n\theta + i\sin n\theta)$

now equating real parts, we get

$\cos n\theta = \cos^n\theta - {}^nC_2\cos^{n-2}\theta\sin^2\theta + {}^nC_4\cos^{n-4}\theta\sin^4\theta + \ldots\ldots$ ...... (1)

Now $(\sin^{2m}\theta) = (\sin^2\theta)^m = (1-\cos^2\theta)^m$

$= (1-q)^m \in Q, \forall$ +ve integer m.

$\cos^{n-2k}$ for +ve integer $k \le \frac{n}{2}$ is a rational number if and only if n is even.

Thus from (1) $\cos n\theta$ is rational number if and only if n is even.

Sol.7 (A)

As $1 + \cot\theta \le \cot\frac{\theta}{2}$ holds

$\Rightarrow$ equality holds when $\cot\frac{\theta}{2} - 1 = 0$

$\Rightarrow \theta = \frac{\pi}{2}$

Sol.8 (A)

We have $\tan\left(\theta + \frac{\pi}{4}\right) = 3\tan 3\theta$

$$\frac{1+\tan\theta}{1-\tan\theta} = 3\left(\frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}\right)$$

$\Rightarrow \frac{1+t}{1-t} = 3\left(\frac{3t-t^3}{1-3t^2}\right)$ where $t = \tan\theta$

$\Rightarrow 3t^4 - 6t^2 + 8t - 1 = 0$

$\Rightarrow t_1 + t_2 + t_3 + t_4 = \frac{0}{3} = 0$

$\Rightarrow \tan\alpha + \tan\beta + \tan\gamma + \tan\delta = 0$

Sol.9 (B)

$\sin^2 12^\circ + \sin^2 21^\circ + \sin^2 39^\circ + \sin^2 48^\circ - \sin^2 9^\circ - \sin^2 18^\circ$

$\Rightarrow \frac{1-\cos 24^\circ}{2} + \frac{1-\cos 42^\circ}{2}$

$+ \sin^2 39^\circ - \sin^2 9^\circ + \sin^2 48^\circ - \sin^2 18^\circ.$

$\Rightarrow \frac{1}{2} - \frac{\cos 24^\circ}{2} + \frac{1}{2} - \frac{\cos 42^\circ}{2}$

$+ \sin 48^\circ \sin 30^\circ + + \sin 30^\circ \sin 66^\circ$

$= 1 - \frac{\cos 24^\circ}{2} - \frac{\cos 42^\circ}{2}$

$+ \frac{1}{2}\sin 48^\circ + \frac{1}{2}\sin 66^\circ$

$= 1 - \frac{\cos 24^\circ}{2} - \frac{\cos 42^\circ}{2}$

$+ \frac{1}{2}(\cos 42^\circ) + \frac{1}{2}\cos 24^\circ$

$= 1$

Sol.10 (B)

We have

$$\left|\sqrt{\frac{1-\sin\alpha}{1+\sin\alpha}} + \sqrt{\frac{1+\sin\alpha}{1-\sin\alpha}}\right|$$

$$= \left|\frac{\cos\frac{\alpha}{2} - \sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2} + \sin\frac{\alpha}{2}} + \frac{\cos\frac{\alpha}{2} + \sin\frac{\alpha}{2}}{\left(\cos\frac{\alpha}{2} - \sin\frac{\alpha}{2}\right)}\right|$$

$$= \left|\frac{2\left(\cos^2\frac{\alpha}{2} + \sin^2\frac{\alpha}{2}\right)}{\cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2}}\right| = \left|\frac{2}{\cos\alpha}\right| = -\frac{2}{\cos\alpha}$$

$\because$ α lies in II quadrant.