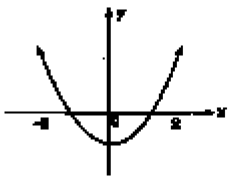
Dear student following is an Easy level [O ● O O O] test paper. Score of 24 Marks in 10 Minutes would be a satisfactory performance. Questions 1-10(+3,–1) (Questions may have more than one option correct)

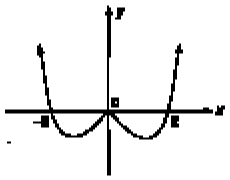
Direction for Q.1 - 6

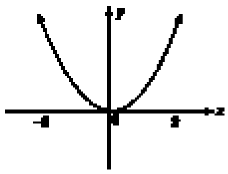
If $f(x) = x^2 - 2|x|$ and

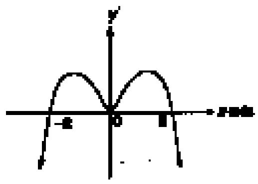
$$g(x) = \begin{cases} \min.\{f(t) : -2 \le t \le x,\ -2 \le x < 0\} \\ \max.\{f(t) : 0 \le t \le x,\ 0 \le x < 3\} \end{cases}, \text{ then}$$

Q.1 The graph of f(x) is :

(A) 

(B) 

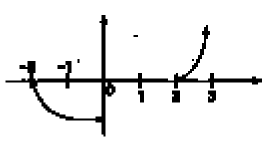
(C) 

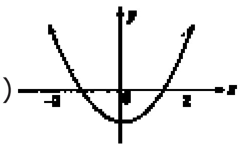
(D) 

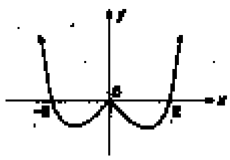
Q.2 The function f(x) is continuous for :
(A) R (B) R – {0}
(C) R – {0, –2, 2} (D) None of these

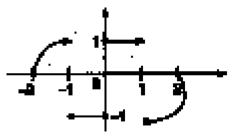
Q.3 The function f(x) is differentiable for, x belong to :
(A) R (B) R – {0}
(C) R – {0, –2, 2} (D) None of these

Q.4 The graph of g(x) is :

(A) 

(B) 

(C) 

(D) 

Q.5 The function g(x) is continuous for :
(A) $x \in R - \{0\}$ (B) $x \in R - \{-1, 0, 1\}$
(C) $x \in R - \{0, 2\}$ (D) None of these

Q.6 The function g(x) is differentiable for :
(A) $x \in R - \{0\}$ (B) $x \in R$
(C) $x \in R - \{-1, 0, 1\}$ (D) None of these

Direction for Q.7 - 8

Let f(x) be defined over an interval for which $x \in R$ given as;

$$f(x) = \int_0^1 |x - t|\, t\, dt, \text{ then}$$

Q.7 The function f(x) is continuous for :
(A) R (B) R – {2/3, 1}
(C) R – {0, 1} (D) None of these

Q.8 The function f(x) is differentiable for :
(A) R – {2/3, 1} (B) R – {0, 1}
(C) R (D) None of these

Direction for Q.9 - 10

Let f be a continuous function on the closed, finite interval [a, b] then (i) f is bounded on [a, b] and (ii) f attains both its max. value M and its min. value m on [a, b] i.e. there is c, d $\in$ [a,b] such that f(c) = M and f(d) = m.

Q.9 Which of the following functions are not bounded

(A) $f(x) = \dfrac{2x}{1 + X^2}$, [–2, 2]

(B) $f(x) = \dfrac{1 - \cos x}{x^2}$, [–2, 2]

(C) $f(x) = \dfrac{x^3 - 8x + 6}{4x + 1}$, [0, 5]

(D) None of these

Q.10 Let $g(x) = 1/x^2$, $x > 0$; $g(0) = 0$ then
(A) g is a continuous function
(B) g is a bounded function
(C) g is bounded on $[1, \infty)$
(D) g has a minimum value on $[1, \infty)$

MATHEMATICS IIT JEE (JULY 1st WEEK CLASS TEST 2) (CONTINIUITY & DIFFERENTIABILITY) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○
										10	○	○	○	○

ANSWER KEY

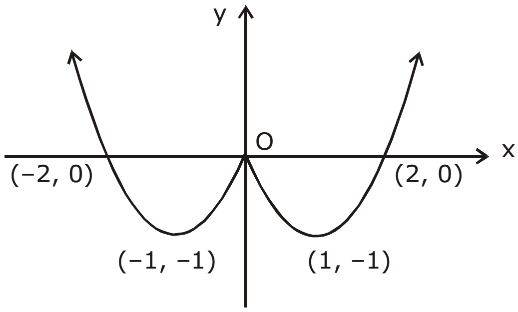
Que.	1	2	3	4	5	6	7	8	9	10
Ans.	B	A	B	A	A	D	A	C	B,D	A,B,D

SOLUTIONS

Sol.1 to 6

1 - (B), 2 - (A), 3 - (B), 4 - (A), 5 - (A), 6 - (D)

The graph of f(x) is show as



It is clear from the curve that f(x) is every where continuous and differentiable except at x = 0.

We know that

If f(x) is an increasing function of [a, b], then

max. $\{f(t) : a \le t \le x, a \le x \le b\} = f(x)$

min. $\{f(t) : a \le t \le x, a \le x \le b\} = f(a)$

and if f(x) is a decreasing function of [a, b] then

max. $\{f(t) : a \le t \le x, a \le x \le b\} = f(a)$

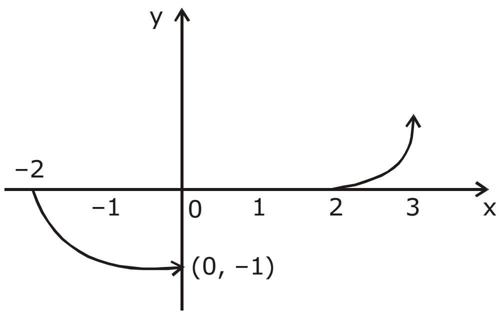
min. $\{f(t) : a \le t \le x, a \le x \le b\} = f(x)$

it is clear from the graph of f(x) that it is decreasing on [–2, –1] and [0, 1] and increasing on [–1, 0] and [1, 3].

Now $$g(x) = \begin{cases} f(x) & \text{for} & -2 \le x \le -1 \\ -1 & \text{for} & -1 \le x < 0 \\ 0 & \text{for} & 0 \le x < 2 \\ f(x) & \text{for} & x \ge 2 \end{cases}$$

$$\Rightarrow \quad g(x) = \begin{cases} x^2 + x & -2 \le x \le -1 \\ -1 & -1 \le x < 0 \\ 0 & 0 \le x < 2 \\ x^2 - 2x & x \ge 2 \end{cases}$$

$\therefore$ graph of g(x) is as



$\therefore$ It is clear that g(x) is discontinuous at x = 0 and non differentiable at x = 0 and x = 2. However it is continuous and differentiable for all other points in (–2, 3).

Sol.7 to 8

7 - (A), 8 - (C)

Here

$$f(x) = \begin{cases} -\int_0^1 (x-t)t\,dt & , \quad x \le 0 \\ \int_0^x (x-t)t\,dt - \int_x^1 (x-t)t\,dt & , \quad 0 < x < 1 \\ \int_0^1 (x-t)t\,dt & , \quad x \ge 1 \end{cases}$$

$$\therefore \quad f(x) = \begin{cases} \dfrac{-3x+2}{6} & , \quad x \le 0 \\ \dfrac{2x^3 - 3x + 2}{6} & , \quad 0 < x < 1 \\ \dfrac{3x-2}{6} & , \quad x \ge 1 \end{cases}$$

Clearly it is continuous at x = 0 and 1, so continuous every where

$$f'(x) = \begin{cases} -\dfrac{1}{2} & , \quad x \le 0 \\ \dfrac{2x^2 - 1}{2} & , \quad 0 < x < 1 \\ \dfrac{1}{2} & , \quad x \ge 1 \end{cases}$$

$\Rightarrow$ f'(x) is also continuous at x = 0 and x = 1.

$\Rightarrow$ f(x) is differentiable everywhere.

Sol.9 (B, D)

f in (i) is continuous on [−2, 2] so is bounded. The maximum value of f on [−2, 2] is 1/2. Again the function (C) is continuous on [0, 5] so bounded. Moreover for f in (iii) $\lim_{x \to 0} f(x) = 1/2$.

Sol.10 (A, B, D)

As $x \to +$, $1/x^2 \to \infty$ so g is not bounded and not continuous on $[0, \infty)$ on the other hand g is bounded on $[1, \infty)$ since $0 \le g(x) \le 1$ but g has not minimum on $[1, \infty)$.