Dear student following is a Moderate level [○ ● ○] test paper. Score of 18 Marks in 15 Minutes would be a satisfactory performance. Questions 1-10 (+3, –1) (All questions have only one option correct)

Q.1 If a is real and the 4th term in the expansion of $\left(ax+\frac{1}{x}\right)^n$ is 5/2, then values of a and n are respectively-

(A) 5, $\frac{1}{2}$ (B) 6, $-\frac{1}{2}$ (C) 3, $\frac{1}{3}$ (D) 6, $\frac{1}{2}$

Q.2 If x^{2k} occurs in the expansion of $\left(x+\frac{1}{x^2}\right)^{n-3}$, then -

(A) n – 2k is a multiple of 2
(B) n – 2k is a multiple of 3
(C) k = 0
(D) None of these

Q.3 Coefficient of the term independent of x in the expansion of

$$\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$$ is-

(A) 210 (B) 105 (C) 70 (D) 35

Q.4 The coefficient of x^k in th eexpansion of E = 1 + (1 + x) + $(1 + x)^2$ + ... + $(1 + x)^n$ is-

(A) ${}^{n}C_{K}$ (B) ${}^{n+1}C_{k}$
(C) ${}^{n+1}C_{k+1}$ (D) None of these

Q.5 In the expansion of $(1 + px)^n$, n $\in$ N, the coefficient of x and x^2 are 8 and 24 respectively, then-

(A) n = 3, p = 2 (B) n = 4, p = 2
(C) n = 4, p = 3 (D) n = 5, p = 3

Q.6 Sum of the last 20 coefficients in the expansion of $(1 + x)^{39}$, when expanded in ascending powers of x, is-

(A) 2^{19} (B) 2^{18}
(C) ${}^{40}C_{20} - 2^{19}$ (D) 2^{38}

Q.7 The largest term in the expansion of $(3 + 2x)^{50}$, where x = 1/5, is-

(A) 5th (B) 6th (C) 8th (D) 9th

Q.8 The coefficient of x^n in the expansion of $\frac{(1+x)^2}{(1-x)^2}$, (|x| < 1), is-

(A) 4n (B) 5n (C) 3n (D) 2n

Q.9 Coefficient of the (r + 1)th term in the expansion of $(1 - 2x)^{-1/2}$ is |x| < 1/2

(A) $\frac{1}{2^r}\ ({}^{2r}C_r)$ (B) ${}^{2r}C_r$
(C) $2^r\ ({}^{2r}C_r)$ (D) None of these

Q.10 If x is nearly equal to 1, then value of $\frac{ax^b - bx^a}{x^b - x^a}$ is nearly equal to-

(A) $\frac{1}{1+x}$ (B) $\frac{1}{1-x}$
(C) $\frac{2}{1+x}$ (D) $\frac{2}{1-x}$

MATHEMATICS IIT JEE (SEPT. 5th WEEK CLASS TEST 2) (BINOMIAL THEOREM) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	⬭	⬭	⬭	⬭	4	⬭	⬭	⬭	⬭	7	⬭	⬭	⬭	⬭
2	⬭	⬭	⬭	⬭	5	⬭	⬭	⬭	⬭	8	⬭	⬭	⬭	⬭
3	⬭	⬭	⬭	⬭	6	⬭	⬭	⬭	⬭	9	⬭	⬭	⬭	⬭
										10	⬭	⬭	⬭	⬭

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	D	B	A	C	B	D	B	A	A	B

SOLUTIONS

Sol.1 (D)

We have

$$T_4 = T_{3+1} = {}^3C_3\ (ax)^{n-3} \left(\frac{1}{x}\right)^3$$

$$= {}^nC_3\ a^{n-3}\ x^{n-6} = \frac{5}{2}$$

$\Rightarrow$ $n - 6 = 0$ and ${}^nC_3\ a^{n-3} = \frac{5}{2}$

$\Rightarrow$ $n = 6$ and ${}^6C_3\ a^3 = \frac{5}{2}$

$\therefore$ $a^3 = \frac{5}{2} \times \frac{3!\,3!}{6!} = \frac{5}{2} \times \frac{1}{20} = \frac{1}{8}$

$\Rightarrow$ $a = \frac{1}{2}$

Thus, $n = 6$, $a = 1/2$.

Sol.2 (B)

$(r + 1)$th term in the expansion of $\left(x + \frac{1}{x^2}\right)^{n-3}$ is given by

$$T_{r+1} = {}^{n-3}C_r\ (x)^{n-3-r} \left(\frac{1}{x^2}\right)^r$$

$$= {}^{n-3}C_r\ x^{n-3-3r}$$

As x^{2k} occurs in the expansion of $\left(x + \frac{1}{x^2}\right)^{n-3}$, we must have $n - 3 - 3r = 2k$ for some non-negative integer r.

$\Rightarrow$ $3(1 + r) = n - 2k$

$\Rightarrow$ $n - 2k$ is a multiple of 3.

Sol.3 (A)

We have

$$x + 1 = (x^{1/3})^3 + 1$$

$$= (x^{1/3} + 1)\ (x^{2/3} - x^{1/3} + 1)$$

$$x - 1 = \left(\sqrt{x} - 1\right)\ \left(\sqrt{x} + 1\right)$$

and $x - x^{1/2} = \sqrt{x}\ (\sqrt{x} - 1)$,

Thus, $\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}}$

$$= x^{1/3} + 1 - (1 + x^{-1/2}) = x^{1/3} - x^{-1/2}$$

Now, $(r + 1)$th term in the expansion of $(x^{1/3} - x^{-1/2})^{10}$ is

$$T_{r+1} = {}^{10}C_r\ (x^{1/3})^{10-r}\ (x^{-1/2})^r\ (-1)^r$$

$$= {}^{10}C_r\ x^{10/3 - 5r/6}\ (-1)^r$$

For this term to be independent of x,

$$\frac{10}{3} - \frac{5r}{6} = 0$$

$\Rightarrow$ $r = \frac{10}{3} \times \frac{6}{5} = 4$

Thus, coefficient of term independent of x is ${}^{10}C_4 = \frac{10!}{4!\,6!} = 210$.

Sol.4 (C)

We have

$E = 1 + (1 + x) + (1 + x)^2 + ... + (1 + x)^n$

$$= \frac{1-(1+x)^{n+1}}{1-(1+x)} = \frac{1}{x}\ [(1 + x)^{n+1} - 1]$$

Thus, coefficient of x^k in E

= coefficient of x^{k+1} in

$$\frac{1}{x}\ [(1 + x)^{n+1} - 1]$$

$$= {}^{n+1}C_{k+1}$$

Sol.5 (B)

We have

$(1 + px)^n = 1 + {}^nC_1(px) + {}^nC_2(px)^2 + ...$

$$= 1 + npx + \frac{1}{2}n(n - 1)\ p^2x^2 + ...$$

According to the hypothesis,

$np = 8$ and $\frac{1}{2}n(n-1)p^2 = 24$

Putting p = 8/n in the second expression, we get

$$\frac{1}{2}n(n-1)\left(\frac{8}{n}\right)^2 = 24$$

$\Rightarrow \frac{n-1}{n} = \frac{24 \times 2}{8 \times 8} = \frac{3}{4}$

$\Rightarrow 4n - 4 = 3n$

$\Rightarrow n = 4$

Putting this value in np = 8, we get p = 2.

Sol.6 (D)

There are 40 terms in the expansion of $(1 + x)^{39}$; Sum of last 20 coefficients is

$S = {}^{39}C_{20} + {}^{39}C_{21} + ... + {}^{39}C_{38} + {}^{39}C_{39}$

$\Rightarrow S = {}^{39}C_{19} + {}^{39}C_{18} + ... + {}^{39}C_1 + {}^{39}C_0$ [Using ${}^nC_r = {}^nC_{n-r}$]

Adding the above two expansions, we get

$2S = {}^{39}C_0 + {}^{39}C_1 + ... + {}^{39}C_{39} = 2^{39}$

$\Rightarrow S = 2^{38}$

Sol.7 (B)

Greatest term in the expansion of $(x + y)^n$ is kth term where $k = \left[\frac{(n+1)y}{x+y}\right]$

In the present case

$$k = \left[\frac{(50+1)(2x)}{3+2x}\right] = \left[\frac{(51)(2/5)}{3+2/5}\right]$$

$$= \left[\frac{102}{17}\right] = 6$$

Thus, 6th term is the largest term.

Sol.8 (A)

We have, for $|x| < 1$

$$\frac{(1+x)^2}{(1-x)^2} = (1 + x)^2 (1 - x)^{-2}$$

$$= (1 + 2x + x^2)(1 + 2x + 3x^2 + 4x^3 + ...)$$

$\therefore$ coefficient of x^n is

$(n + 1) + 2n + (n - 1) = 4n$

Sol.9 (A)

(r + 1)th term in the expansion of $(1 - 2x)^{-1/2}$ is

$$T_{r+1} = \frac{\left(\frac{-1}{2}\right)\left(-\frac{1}{2}-1\right)\left(-\frac{1}{2}-2\right)......\left(-\frac{1}{2}-r+1\right)}{r!}$$

$$= \frac{(-1)^r (1)(3)(5)......(2r-1)}{(2^r)(r!)} (-2)^r x^r$$

$$= \frac{(1)(2)(3)(4).....(2r-1)(2r)}{(2)(4)(6).......(2r)(2^r)r!} (2^r x^r)$$

$$= \frac{(2r)!}{2^r (r!)(r!)} x^r = \frac{1}{2^r} ({}^{2r}C_r) x^r$$

Sol.10 (B)

Let x = 1 + h where h is so small that h^2 and higher powers of h can be neglected. We have

$$\frac{ax^b - bx^a}{x^b - x^a} = \frac{a(1+h)^b - b(1+h)^a}{(1+h)^b - (1+h)^a}$$

$$= \frac{a(1+bh) - b(1+ah)}{(1+bh)-(1+ah)} = \frac{a-b}{(b-a)h}$$

$$= \frac{-1}{h} = \frac{-1}{x-1} = \frac{1}{1-x}$$