Dear student following is a Moderate level [○ ○ ● ○ ○] test paper. Score of 18 Marks in 15 Minutes would be a satisfactory performance. Questions 1-10(+3,–1) (All questions have only one option correct).

Q.1 The number of points in [0, 2] at which $f(x) = |x - 1/2| + |x - 1| + \tan x$ is not finitely differentiable is
(A) 2 (B) 1 (C) 3 (D) 0

Q.2 Let $f(x) = \begin{cases} 3^x & -1 \le x \le 1 \\ 4 - x & 1 < x < 4 \end{cases}$ then at x = 1, f(x) is
(A) continuous (B) differentiable
(C) dis-continuous (D) None of these

Q.3 If $f(x) = \dfrac{\tan\{\pi[x - \pi]\}}{1 + [x]^2}$ where [x] = greatest integer $\le$ x, then
(A) f(x) is discontinuous for some x
(B) continuous at all x but not differentiable for some x
(C) f'(x) exists for all x but f''(x) not
(D) f'(x) exists for all x

Q.4 If $x + 4|y| = 6y$, then y is a function of x such that y is
(A) continuous at x = 0 (B) derivable at x = 0
(C) $\dfrac{dy}{dx} = 3$, r x > 0 (D) None of these

Q.5 The value of a, b for which the function

$$f(x) = \begin{cases} \dfrac{\sin(a+1)x + \sin x}{x} & \text{for } x < 0 \\ b & \text{for } x = 0 \\ \dfrac{(x + 2x^2)^{1/2} - x^{1/2}}{2x^{3/2}} & \text{for } x > 0 \end{cases}$$

is continuous at x = 0 are :
(A) $a = -\dfrac{3}{2}, b = \dfrac{1}{2}$ (B) $a = \dfrac{3}{2}, b = \dfrac{1}{2}$
(C) $a = \dfrac{3}{2}, b = -\dfrac{1}{2}$ (D) None of these

Q.6 The value of K for which the function

$$f(x) = \frac{(e^x - 1)^4}{\sin\left(\dfrac{x^2}{K^2}\right)\log\left(1 + \dfrac{x^2}{2}\right)},\ x \ne 0$$

f(0) = 8
may be continuous function is
(A) 1 (B) –1 (C) ±2 (D) ±3

Q.7 If $f(x) = (1 + ax)^{1/x}$, x < 0
= b, x = 0
$= \dfrac{(x - c)^{1/3} - 1}{(x + 1)^{1/2} - 1}$, x > 0
is continuous atx = 0, then
(A) a = 2/3, b = 2/3, c = 1
(B) $a = \log_e(2/3)$, b = 2/3, c = 1
(C) $a = \log_e(2/3) = b$, c = 1
(D) None of these

Q.8 If f(x) is continuous and f(9/2) = 2/9, then $\lim\limits_{x \to 0} f\left(\dfrac{1 - \cos 3x}{x^2}\right)$ is equal to
(A) 9/2 (B) 2/9 (C) 0 (D) None

Q.9 Let $f(x) = [x] \sin\left(\dfrac{\pi}{[x + 1]}\right)$ where [x] denotes the greatest integer function. The domain of f and pts. of discontinuity of f are
(A) $x \in R : x \notin [-1, 0)$ and $x \in I - \{0\}$
(B) $x \in R : x \in [-1, 0)$ and $x \in I - \{0\}$
(C) $x \in R : x \in (0, -1]$ and $x \in I - \{1\}$
(D) None of these

Q.10 Let $g(x) = \int_0^x f(t)\,dt$, where f is such that $1/2 \le f(t) \le 1$ for $t \in [0, 1]$ and $0 \le f(t) \le 1/2$ for $t \in [1, 2]$. Then the interval in which g(2) lies.
(A) $\left[\dfrac{1}{2}, 1\right]$ (B) $\left[1, \dfrac{3}{2}\right]$ (C) $\left[\dfrac{1}{2}, \dfrac{3}{2}\right]$ (D) None

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MATHEMATICS IIT JEE (JULY 1st WEEK CLASS TEST 4) (CONTINIUITY & DIFFERENTIABILITY) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○
										10	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	C	A	D	A	A	C	B	B	A	C

SOLUTIONS

Sol.1 (C)

$|x - 1/2|$ is not differentiable at $x = 1/2$, $|x - 1|$ is not differentiable at $x = 1$. tanx is not differentiable at $x = \frac{\pi}{2}$ all these lies in $[0, 2]$.

Thus number of points is 3.

Sol.2 (A)

For $-1 \le x \le 1$ $\quad f(x) = 3^x$

$\Rightarrow \quad f(1) = 3^1 = 3$

and $f(1 - 0) = \lim_{h\to\infty} 3^{1-h} = \lim_{h\to\infty} 3.3^{-h} = 3$

for $4 > x > 1$ $\quad f(x) = 4 - x$ then

$f(1 + 0) = \lim_{h\to 0} 4 - (1+h) = \lim_{h\to 0} 3 - h = 3$

Thus $f(1 - 0) = f(1) = f(1 + 0)$

$f(x)$ is continuous at $x = 1$.

Further $\quad Rf'(1) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$

$= \lim_{h\to 0} \frac{4 - (1+h) - 3}{h} = -1$

and $\quad Lf'(1) = \lim_{h\to 0} \frac{f(1-h) - f(1)}{-h}$

$= \lim_{h\to 0} \frac{3^{1-h} - 3}{-h}$

$= \lim_{h\to 0} \frac{3(3^{-h} - 1)}{-h}$

$= \lim_{h\to 0} \frac{3[3^{-h}(-1)\log_e 3 - 0]}{-1}$

$= 3 \log_e 3$ (By L.H. Rule)

$\Rightarrow \quad Lf'(1) \ne Rf'(1)$

Thus $f(x)$ is not differentiable at $x = 1$.

Sol.3 (D)

Since $[x - \pi] = k$ an integer for all real x.

Then $f(x) = \frac{\tan\{\pi[x-\pi]\}}{1+[x]^2} = \frac{\tan k\pi}{1+[x]^2}$

$= 0$ (constant)

since $\tan k\pi = 0$, $1 + [x]^2 \ne 0$

$\because$ A constant function is continuous and differentiable everywhere.

Sol.4 (A)

$\{x + 4|y| = 6y\} = \begin{cases} x + 4y = 6y & \text{if } y \ge 0 \\ x - 4y = 6y & \text{if } y \le 0 \end{cases}$

$\Rightarrow \quad y = \frac{1}{2}x$ if $y \ge 0$ or if $x \ge 0$

and $y = \frac{1}{10}x$ if $y \le 0$ or if $x \le 0$

We observe that

$f(0 + 0) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0} \frac{1}{2}(0+h) = 0$

and $f(0) = 0$ and

$f(0 - 0) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} \frac{1}{10}(0-h) = 0$

$\Rightarrow \quad y = f(x)$ is continuous at $x = 0$.

But $Rf'(0) = \frac{d}{dx}\left(\frac{1}{2}x\right) = \frac{1}{2}$,

$Lf'(0) = \frac{d}{dx}\frac{1}{10}x = \frac{1}{10}$

$\Rightarrow \quad f(x)$ is not derivable at $x = 0$.

Sol.5 (A)

At $\quad x = 0, f(x) = b$

$\Rightarrow \quad f(0) = b.$

$f(0 - 0) = \lim_{h\to 0} f(0-h)$

$= \lim_{h\to 0} \frac{\sin(a+1)(-h) + \sin(-h)}{-h}$

$= \lim_{h\to 0} \left[\frac{\sin(a+1)(h)}{h} + \frac{\sin(h)}{h}\right]$

$= (a + 1) + 1 = a + 2$

and $f(0 + 0) = \lim_{h\to 0} f(0+h)$

$= \lim_{h\to 0} \frac{[(0+h) + 2(0+h)^2]^{1/2} - (0+h)^{1/2}}{2(0+h)^{3/2}}$

$= \lim_{h\to 0} \frac{(h+2h^2)^{1/2} - h^{1/2}}{2h^{3/2}}$

$= \lim_{h\to 0} \frac{h^{1/2}(1+2h)^{1/2} - h^{1/2}}{2h^{3/2}}$

$$= \lim_{h\to 0} \frac{\left(1+\frac{1}{2}.2h+\frac{1}{2}\left(-\frac{1}{2}\right).\frac{1}{2!}2^2h^2+....\right)-1}{2h}$$

$$= \lim_{h\to 0} \frac{1-\frac{1}{2}h+.....}{2}\frac{1}{2} = \frac{1}{2}$$

f(x) will be continuous at x = 0

if f(0) = f(0 – 0) = f(0 + 0)

$\Rightarrow$ b = a + 2 = $\frac{1}{2}$

$\Rightarrow$ a = $-\frac{3}{2}$, b = $\frac{1}{2}$

Sol.6 (C)

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{\left(\frac{e^x-1}{x}\right)^4.x^4}{\frac{\sin\left(\frac{x^2}{K^2}\right)}{\frac{x^2}{K^2}}.\frac{x^2}{K^2}\log\left(1+\frac{x^2}{2}\right)}$$

$$= \lim_{x\to 0} \frac{K^2x^2}{\log\left(1+\frac{x^2}{2}\right)}$$

$$= K^2 \lim_{x\to 0} \frac{1}{\frac{1}{x^2}\log\left(1+\frac{x^2}{2}\right)}$$

$$= 2K^2. \lim_{x\to 0} \frac{1}{\frac{2}{x^2}\log\left(1+\frac{x^2}{2}\right)}$$

$= 2K^2.1 = 2K^2$

Since f is cont. $\therefore$ it is cont. at x = 0

$\therefore$ f(0) = $\lim_{x\to 0} f(x)$ $\Rightarrow$ 8 = $2K^2$

$K^2 = 4$ $\therefore$ K = ±2.

Sol.7 (B)

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0}(1+ax)^{1/x} = e^a$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0} \frac{(x+c)^{1/3}-1}{(x+1)^{1/2}-1}$$

$$\lim_{x\to 0} \frac{\frac{1}{3}(x+c)^{-2/3}}{\frac{1}{2}(x+1)^{-1/2}} = \frac{2}{3}c^{-2/3}$$

Since f(x) is cont. at x = 0

$\Rightarrow$ $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0) = b$

$\therefore$ $e^a = \frac{2}{3}c^{-2/3} = b.$

If c = 1, b = $\frac{2}{3}$, a = $\log\left(\frac{2}{3}\right)$.

Sol.8 (B)

$$\lim_{x\to 0} f\left(\frac{1-\cos 3x}{x^2}\right) = f\left(\lim_{x\to 0}\frac{1-\cos 3x}{x^2}\right)$$

$$= f\left(\lim_{x\to 0}\frac{2\sin^2\frac{3x}{2}}{x^2}\right)$$

$$= f\left(\lim_{x\to 0} 2.\left(\frac{\sin\frac{3x}{2}}{\frac{3x}{2}}\right)^2.\frac{9}{4}\right) = f\left(\frac{9}{2}\right) = \frac{2}{9}$$

Hence $\lim_{x\to 0} f\left(\frac{1-\cos 3x}{x^2}\right) = \frac{2}{9}$

Sol.9 (A)

[x + 1] = 0 if 0 ≤ x + 1 < 1

i.e. if −1 ≤ x < 0

$\therefore$ D_f = R – [−1, 0)

Now $\sin\left(\frac{\pi}{[x+1]}\right)$ is continuous at all pts. of R — [−1, 0) and [x] is continuous on R – I, where I is the set of integers.

Thus the pts. when f can possibly be discontinuous are −3, −2, −1, 0, 1, 2, 3 ...

But for 0 ≤ x < 1, [x] = 0

and $\sin\left(\frac{\pi}{[x+1]}\right)$ is defined.

$\therefore$ f(x) = 0 for 0 ≤ x < 1

Also f(x) is not defined on −1 ≤ x < 0

∴ continuity of f at '0' means, continuity of f from the right at 0. Since f is cont. of '0' from the right.

∴ f is cont. at 0.

Hence set of pts. of discontinuities is I − {0}.

Sol.10 (C)

$$g(x) = \int_0^x f(t)dt$$

$$\Rightarrow \quad g(2) = \int_0^2 f(t)dt$$

$$\Rightarrow \quad g(2) = \int_0^1 f(t)dt + \int_1^2 f(t)dt \quad ....(i)$$

Now, $\frac{1}{2} \le$ **f(t)** $\le$ **1 for t** $\in$ **[0, 1]**

we get $\int_1^2 \frac{1}{2}dt \le \int_0^1 f(t)\,dt \le \int_0^1 1\,dt$

$$\frac{1}{2} \le \int_0^1 f(t)\,dt \le 1 \quad ....(ii)$$

Now, **0** $\le$ **f(t)** $\le \frac{1}{2}$ **for t** $\in$ **[1, 2]**

$$\Rightarrow \quad \int_1^2 0\,dt \le \int_1^2 f(t)\,dt \le \int_1^2 \frac{1}{2}\,dt$$

$$0 \le \int_1^2 f(t)\,dt \le \frac{1}{2} \quad ....(iii)$$

from (ii) and (iii), we get

$$0 + \frac{1}{2} \le \int_0^1 f(t)\,dt + \int_1^2 f(t)\,dt \le 1 + \frac{1}{2}$$

or $\frac{1}{2} \le g(2) \le \frac{3}{2}$

$$\Rightarrow \quad g(2) \in \left[\frac{1}{2}, \frac{3}{2}\right]$$