Dear student following is an Easy level [○ ● ○ ○ ○] test paper. Score of 18 Marks in 10 Minutes would be a satisfactory performance. Questions 1-8(+3,–1) (Questions may have more than one option correct)

Direction for Questions (1 - 3)

Let $f(x) = \begin{cases} x + a, & \text{if } x < 0 \\ |x - 1|, & \text{if } x \ge 0 \end{cases}$

and $g(x) = \begin{cases} x + 1, & \text{if } x < 0 \\ (x - 1)^2 + b, & \text{if } x \ge 0 \end{cases}$

where 'a' and 'b' are non-negative numbers, then

Q.1 The composite function (gof)(x), is :

(A) $g(f(x)) = \begin{cases} x+a+1 & , & x < -a \\ (x+a-1)^2+b & , & -a \le x \le 0 \\ x^2+b & , & 0 \le x < 1 \\ (x-2)^2+b & , & x \ge 1 \end{cases}$

(B) $g(f(x)) = \begin{cases} x+a-1 & , & x < -a \\ (x+a-1)^2+b & , & -a \le x < 0 \\ x^2+b & , & 0 \le x < 1 \\ (x+2)^2+b & , & 1 \ge x \end{cases}$

(C) $g(f(x)) = \begin{cases} x-a+1 & , & x < -a \\ (x-a+1)^2+b & , & -a \le x < 0 \\ x^2+b & , & 0 \le x < 1 \\ (x-2)^2+b & , & x \ge 1 \end{cases}$

(D) $g(f(x)) = \begin{cases} x+a+1 & , & x < a \\ (x+a-1)^2+b & , & x \ge 0 \\ (x-2)^2+b & , & x < 0 \end{cases}$

Q.2 If (gof) (x) is continuous for all real x, then the value of 'a' and 'b' are :
(A) 1, 0 (B) –1, 0 (C) 2, 3 (D) –1, R

Q.3 If (gof) (x) is differentiable for all real x, then the value of 'a' and 'b' are :
(A) 1, 0 (B) –1, 0 (C) 2, 3 (D) –1, R

Direction for Questions (4 - 5)

Let $f : R \to R$ be a function defined as,

$f(x) = \begin{cases} 1-|x| & , & |x| \le 1 \\ 0 & , & |x| > 1 \end{cases}$

and $g(x) = f(x - 1) + f(x + 1)$, $\forall\, x \in R$. Then

Q.4 The value of g(x) is :

(A) $g(x) = \begin{cases} 0 & , & x \le -3 \\ 2+x & , & -3 \le x \le -1 \\ -x & , & -1 < x \le 0 \\ x & , & 0 < x \le 1 \\ 2-x & , & 1 \le x \le 3 \\ 0 & , & x > 3 \end{cases}$

(B) $g(x) = \begin{cases} 0 & , & x \le -2 \\ 2+x & , & -2 \le x \le -1 \\ -x & , & -1 < x \le 0 \\ x & , & 0 < x \le 1 \\ 2-x & , & 1 < x \le 2 \\ 0 & , & x > 2 \end{cases}$

(C) $g(x) = \begin{cases} 0 & , & x \le 0 \\ 2+x & , & 0 < x < 1 \\ -x & , & 1 \le x \le 2 \\ x & , & 2 < x < 3 \\ 2-x & , & 3 \le x < 4 \\ 0 & , & 4 \le x \end{cases}$

(D) None

Q.5 The function g(x) is continuous for, $x \in$
(A) R-{0,1,2,3,4} (B) R-{–2,–1,0,1,2}
(C) R (D) None of these

Q.6 The function g(x) is differentiable for, $x \in$
(A) R (B) R-{–2,–1,0,1,2}
(C) R-{0,1,2,3,4} (D) None of these

Q.7 If $f(x) = \begin{cases} 3^x & , & -1 \le x \le 1 \\ 4-x & , & 1 < x < 4 \end{cases}$, then f(x) is
(A) continuous at x = 1
(B) differentiable at x = 1
(C) not differentiable at x = 1
(D) discontinuous at x = 1

Q.8 If $f(x) = \sum_{r=1}^{n} a_r |x|^r$, where a_i's are real constant, then f(x) is
(A) continuous at x = 0 for all a_i
(B) differentiable at x = 0 for all $a_i \in R$
(C) differentiable at x = 0 for all $a_{2k+1} = 0$.
(D) None of these

MATHEMATICS IIT JEE (JULY 1st WEEK CLASS TEST 5) (CONTINIUITY & DIFFERENTIABILITY) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	⬭	⬭	⬭	⬭	4	⬭	⬭	⬭	⬭	7	⬭	⬭	⬭	⬭
2	⬭	⬭	⬭	⬭	5	⬭	⬭	⬭	⬭	8	⬭	⬭	⬭	⬭
3	⬭	⬭	⬭	⬭	6	⬭	⬭	⬭	⬭					

ANSWER KEY

Que.	1	2	3	4	5	6	7	8
Ans.	A	A	A	B	C	B	A,C	A,C

SOLUTIONS

Sol. (1 - 3)

1 - (A), 2 - (A), 3 - (A)

Clearly,

$$g(f(x)) = \begin{cases} x+a+1 & , \quad x<-a \\ (x+a-1)^2+b & , \quad -a \le x < 0 \\ x^2+b & , \quad 0 \le x < 1 \\ (x-2)^2+b & , \quad x \ge 1 \end{cases}$$

$$\left.\begin{matrix} g(f(-a)^-) = 1 \\ g(f(-a)^+) = 1+b \end{matrix}\right\} \Rightarrow b = 0$$

Now, $\left.\begin{matrix} g(f(0)^-) = (a-1)^2 \\ g(f(0)^+) = 0 \end{matrix}\right\} \Rightarrow a = 1$

For this value of 'a' and 'b', g(f(x)) is continuous.

Now, $g(f(x)) = \begin{cases} x+2 & , \quad x<-1 \\ x^2 & , \quad -1 \le x < 1 \\ (x-2)^2 & , \quad x \ge 1 \end{cases}$

Clearly, at x = 0, g(f(x)) is differentiable.

Sol. (4 - 6)

4 - (B), 5 - (C), 6 - (B)

Given function f(x) can be rewritten as,

$$f(x) = \begin{cases} 0 & , \quad x<-1 \\ 1+x & , \quad -1 \le x \le 0 \\ 1-x & , \quad 0 < x \le 1 \\ 0 & , \quad x > 1 \end{cases}$$

$\Rightarrow \quad f(x-1) = \begin{cases} 0 & , \quad x-1<-1 \\ 1+(x-1) & , \quad -1 \le x-1 \le 0 \\ 1-(x-1) & , \quad 0 < x-1 \le 1 \\ 0 & , \quad x-1 > 1 \end{cases}$

or $\quad f(x-1) = \begin{cases} 0 & , \quad x<0 \\ x & , \quad 0 \le x \le 1 \\ 2-x & , \quad 1 < x \le 2 \\ 0 & , \quad x > 2 \end{cases}$

Also, $f(x+1) = \begin{cases} 0 & , \quad x+1<-1 \\ 1+(x+1) & , \quad -1 \le x+1 \le 0 \\ 1-(x+1) & , \quad 0 < x+1 \le 1 \\ 0 & , \quad x+1 > 1 \end{cases}$

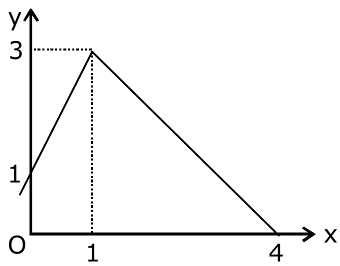
or $\quad f(x+1) = \begin{cases} 0 & , \quad x<-2 \\ 2+x & , \quad -2 \le x \le -1 \\ -x & , \quad -1 < x \le 0 \\ 0 & , \quad x > 0 \end{cases}$

Now g(x) = f(x − 1) + f(x + 1)

$$= \begin{cases} 0 & , \quad x<-2 \\ 2+x & , \quad -2 < x \le -1 \\ -x & , \quad -1 < x \le 0 \\ x & , \quad 0 < x \le 1 \\ 2-x & , \quad 1 < x \le 2 \\ 0 & , \quad x > 2 \end{cases}$$

It is easy to check that g(x) is continuous for all $x \in R$ and non-differentiable at x = −2, −1, 0, 1, 2.

Sol.7 (A, C)



Here

$$f(x) = \begin{cases} 3^x & , \quad -1 \le x \le 1 \\ 4-x & , \quad 1 < x < 4 \end{cases}$$

from the graph itis clear that it is continuous for all x in [−1, 4) and not differentiable at x = 1.

Sol.8 (A, C)

We know that $|x|^r$, r = 0, 1, 2, ... are all continuous every where.

$\therefore \quad f(x) = \sum_{r=1}^{n} a_r |x|^r$ is every where continuous

Since, $|x|$, $|x|^3$, $|x|^5$, ... are not differentiable at x = 0

whereas $|x|^2$, $|x|^4$, ... are every where differentiable.

$\therefore \quad f(x) = \sum_{r=1}^{n} a_r |x|^r$ is not differentiable at x = 0, if any one of a_1, a_3, a_5, ... is non-zero.

Thus, for f(x) to be differentiable at x = 0, we must have $a_1 = a_3 = a_5 .... = 0$

i.e., $a_{2k+1} = 0$.