Dear student following is an Easy level [O ● O O O] test paper. Score of 18 Marks in 10 Minutes would be a satisfactory performance. Questions 1-8(+3,–1) (Questions may have more than one option correct)

Q.1 If $f(x) = \lim_{n\to\infty}\left(\sin\frac{\pi x}{2}\right)^{2n}$, then f(x) is discontinuous at x =

(A) 2n (B) 2n + 1

(C) n (D) None of these

Q.2 If $f(x) = \dfrac{\sin 2x + A\sin x + B\cos x}{x^3}$ is continuous at x = 0, then the values of A and B are

(A) −2, 0 (B) 2, 0

(C) −2, 1 (D) 2, − 1

Q.3 Let $f(x) = [x] \sin\left[\frac{\pi}{x+1}\right]$ where [ ] denotes the greatest integer function. The points of discontinuity of f(x) in the domain are-

(A) $x \in I$ (B) $x \in I - \{-1\}$

(C) $x \in I - \{1\}$ (D) None of these

Q.4 Which of the following functions are continuous on (0, 1)

(A) $x - [x]$ (B) $\dfrac{1}{x-[x]}$

(C) $(-1)^{[x]}$ (D) $\sin [x]$

PASSAGE :

The continuity on an interval has a geometric interpretation, namely, a function f defined on an interval I is continuous on I if its graph has no 'holes' or 'jumps, f is said to have a removable discontinuity at c if f(x) has a limit at c but $\lim_{x\to c} f(x) \neq f(c)$. If $\lim_{x\to c+} f(x)$ and $\lim_{x\to c-} f(x)$ exist but are not equal then c is called jump discontinuity. If $\lim_{x\to c+} f(x)$ or $\lim_{x\to c-} f(x)$ fail to exist then c is called infinite discontinuity.

Q.5 Let f(x) = sgn x then x = 0

(A) Is a point of continuity

(B) Is a jump discontinuity

(C) Is a removable discontinuity

(D) Is an infinite discontinuity

Q.6 Let $g(x) = \begin{cases} x^2+5 & , \; x<2 \\ 10 & , \; x=2 \\ 1+x^3 & , \; x>2 \end{cases}$

then x = 2 is-

(A) A point of continuity

(B) Is a removable discontinuity

(C) Is a jump discontinuity

(D) Is an infinite discontinuity

Q.7 Let g(x) = x + 7, x < − 3; g(x) = |x − 2|, − 3 ≤ x < − 1; $g(x) = x^2 - 2x$, − 1 ≤ x < 3 and g(x) = 2x − 3, x ≥ 3, then-

(A) x = − 1 is a jump discontinuity

(B) x = 3 is an infinite discontinuity

(C) x = − 3 is a jump discontinuity

(D) x = − 1 is a removable discontinuity

Q.8 $g(x) = \begin{cases} 1 & , \; x \le -2 \\ \frac{1}{2}x & , \; -2 < x < 4 \\ \sqrt{x} & , \; x \ge 4 \end{cases}$

then

(A) g is a continuous function

(B) All the discontinuities are removable discontinuities

(C) All the discontinuities are jump

(D) All the discontinuities are infinite.

MATHEMATICS IIT JEE (JULY 1st WEEK CLASS TEST 1) (CONTINIUITY & DIFFERENTIABILITY) ANSWER KEY

Name : .... **Roll No. :** ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	O	O	O	O	4	O	O	O	O	7	O	O	O	O
2	O	O	O	O	5	O	O	O	O	8	O	O	O	O
3	O	O	O	O	6	O	O	O	O					

ANSWER KEY

Que.	1	2	3	4	5	6	7	8
Ans.	B	A	B	All	B	B	C	C

SOLUTIONS

Sol.1 (B)

$$\lim_{n\to\infty} x^{2n} = \begin{cases} 0, & |x| < 1 \\ 1, & |x| = 1 \end{cases}$$

$\therefore$ $f(x) = \lim_{n\to\infty} \left(\sin\frac{\pi x}{2}\right)^{2n}$

$$= \begin{cases} 0, & \left|\sin\frac{\pi x}{2}\right| < 1 \\ 1, & \left|\sin\frac{\pi x}{2}\right| = 1 \end{cases}$$

Thus f(x) is continuous for all x, except for those values of x for which $\left(\sin\frac{\pi x}{2} = 1\right)$ i.e. x is odd integer.

$\Rightarrow$ x = (2n + 1), where $x \in I$

L.H.L. = $\lim_{x\to 2n+1} f(x) = 0$ .... (1)

and f(2n + 1) = 1 .... (2)

$\Rightarrow$ L.H.L. $\neq$ f(2n + 1)

or f(x) is discontinuous at x = (2n + 1))

Sol.2 (a)

As f(x) is continuous at x = 0,

$f(0) = \lim_{x\to a} f(x)$ and both f(0) and $\lim_{x\to a} f(x)$ are finite.

$\Rightarrow$ $f(0) = \lim_{x\to 0} \frac{\sin 2x + A\sin x + B\cos x}{x^3}$

As Denominator $\to$ 0 as x $\to$ 0,

$\therefore$ Numerator should also $\to$ 0 as x $\to$ 0.

Which is possible only if (for f(0) to be finite).

sin 2(0) + A sin (0) + B cos 0 = 0.

$\Rightarrow$ B = 0.

$\therefore$ $f(0) = \lim_{x\to 0} \frac{\sin 2x + A\sin x}{x^3}$

$\Rightarrow$ $f(0) = \lim_{x\to 0} \left(\frac{\sin x}{x}\right)\left(\frac{2\cos x + A}{x^2}\right)$

$= \lim_{x\to 0} \frac{2\cos x + A}{x^2}$

Again we can see that Denominator $\to$ 0 as x $\to$ 0,

$\therefore$ Numerator should also approach 0 as x $\to$ 0 (for f(0) to be finite).

$\Rightarrow$ 2 + A = 0 $\Rightarrow$ A = – 2

Sol.3 (B)

As greatest integer function is discontinuous at integer points, f(x) is continuous for all non-integer points.

Checking continuity at x = a (where a – I)

L.H.L. = $\lim_{h\to 0}$ [a – h] sin $\left(\frac{\pi}{[a+1-h]}\right)$

$\Rightarrow$ L.H.L. = (a – 1) sin $\left(\frac{\pi}{a}\right)$ ....(1)

R.H.L. = $\lim_{h\to 0}$ [a + h] sin $\left(\frac{\pi}{[a+1+h]}\right)$

$\Rightarrow$ R.H.L. = a sin $\left(\frac{\pi}{a+1}\right)$ .....(2)

From (1) and (2), L.H.L. $\neq$ R.H.L.

$\Rightarrow$ f(x) is discontinuous at x = a

(i.e. at integral values of x)

So points of discontinuity are $x \in I \cap D$.

(i.e. integers lying in the set of domain)

$\Rightarrow$ $x \in I - \{-1\}$.

Sol.4 (All)

For c $\in$ (0, 1), [c] = 0, so x – [x] = x, $\frac{1}{x-[x]} = \frac{1}{x}$, (–1) [x] = 1 and sin [x] = 0 for all x $\in$ (0, 1). Hence all the function in (A), (B), (C), (D) are continuous functions on (0, 1)

Sol.5 (B)

$$\text{sgn } x = \begin{cases} 1 & , \ x > 0 \\ 0 & , \ x = 0 \\ -1 & , \ x < 0 \end{cases} \quad \lim_{x \to 0+} f(x) = 1 \text{ and } \lim_{x \to 0-}$$

$f(x) = -1$

Thus x = 0 is a jump discontinuity.

Sol.6 (B)

$\lim_{x \to 2+} f(x) = 1 + 2^3 = 9;$

$\lim_{x \to 2-} f(X) = 4 + 5 = 9$

so $\lim_{x \to 2} f(x)$ exists but is not equal to f(2) = 10

Thus x = 2 is a removable discontinuity.

Sol.7 (C)

$\lim_{x \to -1+} f(x) = \lim_{x \to -1+} x^2 - 2x = 3,$

$\lim_{x \to -1-} f(x) = \lim_{x \to -1} |x - 2| = 3$

so $\lim_{x \to -1} f(x)$ exists and is equal to f(–1) = 3.

Thus f is continuous at x = – 1.

$\lim_{x \to -3+} f(x) = \lim_{x \to -3+} |x - 2| = 5,$

$\lim_{x \to -3-} f(x) = \lim_{x \to -3-} x + 7 = 4.$

So x = – 3 is a jump discontinuity.

Sol.8 (C)

$\lim_{x \to 4+} g(x) = \lim_{x \to 4-} g(x) = 2$

so f is continuous at x = 4.

$\lim_{x \to -2-} g(x) = 1$ but $\lim_{x \to -2+} g(x) = -1$

so x = – 2 is a jump discontinuity and there is no other discontinuity.