Dear student following is a Moderate level [O O ● O O] test paper. Score of 18 Marks in 10 Minutes would be a satisfactory performance. Questions 1-10(+3,–1) (All questions have only one option correct)

Q.1 If $f(x) = p\,|\sin x| + qe^{|x|} + r|x|^3$ and if f(x) is differentiable at x = 0, then-
(A) q + r = 0, p is any real number
(B) p + q = 0; r is any real number
(C) q = 0; r = 0; p is any real number
(D) r = o; p = 0; q is any real number

Q.2 If f(x) is differentiable everywhere then-
(A) |f(x)| is differentiable everywhere
(B) $|f|^2$ is differentiable everywhere
(C) f|f| is not differentiable at some point
(D) None of these

Q.3 Suppose f is differentiable at x = 1 and $\lim_{h\to 0} \frac{1}{h} f(1 + h) = 5$ then-
(A) f'(1) = 4 (B) f'(1) = 3
(C) f'(1) = 6 (D) None of these

Q.4 Let f : R $\to$ R and g : R $\to$ R be defined by g(x) = xf(x) then-
(A) g is a differentiable function
(B) g is differentiable at 0 if f is continuous at 0
(C) g is one-one if f is one-one
(D) None of these

Q.5 If $f(x) = \sqrt{x + 2\sqrt{2x - 4}} + \sqrt{x - 2\sqrt{2x - 4}}$ then-
(A) f is differentiable at all points of its domain except x = 4
(B) f is differentiable on $(2, \infty)$
(C) f is differentiable on $(-\infty, \infty)$
(D) f'(x) = 0 for all $x \in [2, 6)$

Q.6 Number of points of non-differentiability of f(x) = max. {sin x, cos x, 0} in $(0, 2n\pi)$ is-
(A) 3 (B) 3n (C) 2 (D) 2n

Q.7 Let f and g be differentiable functions satisfying g'(a) = 2, g(a) = b and fog = I (identity function). Then f'(b) is equal to-
(A) 2 (B) $\frac{2}{3}$
(C) $\frac{1}{2}$ (D) None of these

Q.8 If f(x) = x + tan x and g(x) is the inverse of f(x) then g'(x) is equal to :
(A) $\frac{1}{1 + (g(x) - x)^2}$ (B) $\frac{1}{2 + (g(x) + x)^2}$
(C) $\frac{1}{2 + (g(x) - x)^2}$ (D) None of these

Q.9 If $f(x) = x\left[\sqrt{x} - \sqrt{(x + 1)}\right]$, then
(A) f(x) is continuous but not differentiable at x = 0
(B) f(x) is continuous and differentiable at x = 0
(C) f(x) is not differentiable at x = 0
(D) None of these

Q.10 Let $f(x) = x - |x - x^2|,\ x \in [-1, 1]$. Then the number of points at which f(x) is discontinuous is-
(A) 0 (B) 1 (C) 2 (D) None

MATHEMATICS IIT JEE (JULY 2nd WEEK CLASS TEST 1) (CONTINIUITY & DIFFERENTIABILITY) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	⬭	⬭	⬭	⬭	4	⬭	⬭	⬭	⬭	7	⬭	⬭	⬭	⬭
2	⬭	⬭	⬭	⬭	5	⬭	⬭	⬭	⬭	8	⬭	⬭	⬭	⬭
3	⬭	⬭	⬭	⬭	6	⬭	⬭	⬭	⬭	9	⬭	⬭	⬭	⬭
										10	⬭	⬭	⬭	⬭

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	B	B	D	B	A	B	C	C	B	A

SOLUTIONS

Sol.1 (B)

For $-\frac{\pi}{2} < x \le 0$, $f(x) = -p\sin x + qe^{-x} - rx^3$,

so $f'(0-) = \lim_{x\to 0-} \frac{f(x) - f(0)}{x - 0}$

$= \lim_{x\to 0-}\left[-\frac{p\sin x}{x} - q\left(\frac{e^{-x}-1}{-x}\right) - rx^2\right]$

$= -p - q$

For $0 < x < \frac{\pi}{2}$, $f(x) = p\sin x + qe^{x} + rx^3$,

$f'(0+) = \lim_{x\to 0+} \frac{f(x) - f(0)}{x - 0}$

$= \lim_{x\to 0+}\left[\frac{p\sin x}{x} + q\left(\frac{e^{x}-1}{x}\right) - rx^2\right]$

$= p + q$. For f to be differentiable at $x = 0$, we must have $p + q = -p - q$

$\Rightarrow\ p + q = 0$.

Sol.2 (B)

If f is differentiable then $|f|^2(x) = f(x)^2$ which is differentiable. For $f(x) = x$, (A) and (C) do not hold.

Sol.3 (D)

Since f is differentiable so it is continuous also. Thus

$f(1) = \lim_{h\to 0} f(1 + h)$

$= \lim_{h\to 0} h\frac{f(1+h)}{h} = (0)(5) = 0$

Hence $f'(1) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$

$= \lim_{h\to 0} \frac{f(1+h)}{h} = 5$

Sol.4 (B)

Take $f(x) = \text{sgn } x$ then $g(x) = |x|$, hence g need not be differentiable. Take $f(x) = x$, then $g(x) = x^2$ which is not one-one.

Let f be continuous at 0, then for $h \ne 0$.

$\frac{g(0+h) - g(0)}{h} = \frac{hf(h)}{h} = f(h)$.

Therefore, $g'(0) = f(0)$.

Sol.5 (A)

The domain of f is $[2, \infty)$. Put $t = \sqrt{2x-4}$

$f(x) = \sqrt{t^2/2 + 2 + 2t} + \sqrt{t^2/2 + 2 - 2t}$

$= \frac{1}{\sqrt{2}}(t + 2) + \frac{1}{\sqrt{2}}|t - 2|$

$= \begin{cases} \frac{1}{\sqrt{2}} \times 4 & \text{if } t < 2 \\ \sqrt{2}t & \text{if } t \ge 2 \end{cases}$

$= \begin{cases} 2\sqrt{2} & \text{if } x \in (2,4) \\ 2\sqrt{x-2} & \text{if } x \in (4,\infty) \end{cases}$

Hence $f'(x) = \begin{cases} 0 & \text{if } x \in (2,4) \\ \frac{1}{\sqrt{x-2}} & \text{if } x \in (4,\infty) \end{cases}$

and f'(4) does nto exist

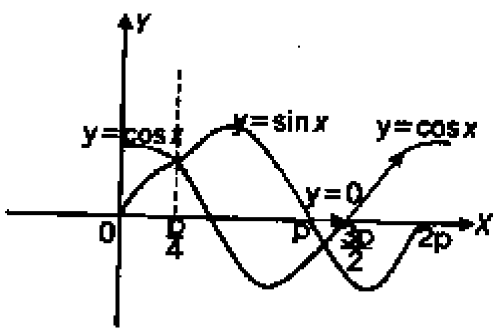
Sol.6 (B)

Here, we know sin x and cos x are periodic with period 2π. Thus we could sketch the curve as; (In the interval 0 to 2π)

Which shows,

y = max. {sin x, cos x, 0}

$= \begin{cases} \cos x, & 0 < x < \frac{\pi}{4} \text{ or } \frac{3\pi}{2} < x < 2\pi \\ 0, & \pi < x < \frac{3\pi}{2} \\ \sin x, & \frac{\pi}{4} < x < \pi \end{cases}$



Clearly, y = max. {sin x, cos x, 0} is not differentiable at 3 points when $x = (0, 2\pi)$

Thus, y = max. {sin x, cos x, 0} is not differentiable at 3n points.

Sol.7 (C)

We have, fog = I

$\Rightarrow$ f{g(x)} = x, for all $x \in R$

$\therefore$ f'{g(x)}.g'(x) = 1, for all $x \in R$

$$\Rightarrow f'(g(a)) = \frac{1}{g'(a)}$$

$$\Rightarrow f'(b) = \frac{1}{2}$$

Sol.8 (C)

We have f(x) = x + tan x

$\Rightarrow f(f^{-1}(x)) = f^{-1}(x) + \tan(f^{-1}(x))$

$\Rightarrow x = g(x) + \tan(g(x))$ ...... (1)

$= g'(x) + \sec^2(g(x)).g'(x)$

$\{\because g(x) = f^{-1}(x)\}$

$$\Rightarrow g'(x) = \frac{1}{1+\sec^2(g(x))}$$

$$\Rightarrow g'(x) = \frac{1}{2+\tan^2(g(x))}$$

$$\Rightarrow g'(x) = \frac{1}{2+(x-g(x))^2}$$

Sol.9 (B)

It is easy to see that

f(0 + 0) = f(0 − 0) = f(0) = 0

(This is left for the students).

Hence f(x) is continuous at x = 0.

Now

$$Lf'(0) = \lim_{h\to 0} \frac{(0-h)\left[\sqrt{(0-h)} - \sqrt{(0-h+1)}\right] - 0}{-h}$$

$$= \lim_{h\to 0} \left[\sqrt{(-h)} - \sqrt{(-h+1)}\right]$$

$$= 0 - \sqrt{1} = -1$$

and $$Rf'(0) = \lim_{h\to 0} \frac{(0+h)\left[\sqrt{(0+h)} - \sqrt{(0+h+1)}\right] - 0}{h}$$

$$= \lim_{h\to 0} \left[\sqrt{(h)} - \sqrt{(h+1)}\right]$$

$$= 0 - \sqrt{1} = -1$$

Since Lf'(0) = Rf'(0) = − 1, the function f(x) is differentiable at x = 0.

Sol.10 (A)

f(x) = x − |x| |1 − x|

Now each of x, |x| and |1 − x| is continuous function and we know that the product and algebraic sum of continuous functions is again a continuous function.

Hence there is no point of discontinuity.