Dear student following is a Moderate level [○ ○ ● ○ ○] test paper. Score of 18 Marks in 10 Minutes would be a satisfactory performance. Questions 1-10(+3,–1) (Questions may have more than one correct option)

Q.1 $f(a)\,g(b) - g(a)\,f(b) = (b - a)\{f(a)\,g'(c) - g(a)\,f'(c)\}$, where $a < c < b$ then
(A) f(x), g(x) both are continous in [a, b]
(B) f(x), g(x) are not continous in [a, b]
(C) f(x), g(x) both are differentiable in (a, b)
(D) f(x), g(x) are not differentiable in (a, b)

Q.2 Let f(x) be a continuous function and satisfies

$$\begin{cases} f^3(x) - 5f^2(x) + 10f(x) - 12 \le 0 \\ f^2(x) - 4f(x) + 3 \ge 0 \\ f^2(x) - 6f(x) + 8 \le 0 \end{cases}$$

If the +ve numbers b_1, b_2, b_3 are in G.P. then $f(1) + \ell n\, b_1, f(2) + \ell n\, b_2, f(3) + \ell n\, b_3$ are in
(A) A.P. (B) G.P. (C) H.P. (D) None

Q.3 If $f(x) = \min\{1, x^2, x^3\}$, then
(A) f(x) is continouous $\forall\, x \in R$
(B) $f'(x) > 0, \forall\, x \in R$
(C) f(x) is not differentiable but continuous $\forall\, x \in R$
(D) f(x) is not differentiable for two values of x.

Q.4 Let $f(x) = |\sin x|$, then the function f(x) is
(A) continuous and differentiable
(B) discontinuous
(C) continuous and not differentiable
(D) None of these

Q.5 $f(x) = \begin{cases} x^m \sin\left(\frac{1}{x}\right) & , \quad x \ne 0 \\ 0 & , \quad x = 0 \end{cases}$,

then f(x) is differentiable at x = 0 if
(A) $m > 0$ (B) $m \le 0$
(C) $0 < m \le 1$ (D) None of these

Q.6 If $f(x) = px^2 - q, \quad x \in [0, 1]$
$\quad\;\;= x + 1, \quad x \in [1, 2]$
and f(1) = 2, then the value of the pair (p, q) for which f(x) cannot be continuous at x = 1 is .....
(A) (2, 0) (B) (1, –1)
(C) (4, 2) (D) (1, 1)

Q.7 Let f(x) be a continuous function defined on [1, 3]. If f(x) takes rational values for all x and f(2) = 10, then the value of f(1.5) is .....
(A) 7.5 (B) 10 (C) 5 (D) None

PASSAGE :
Consider the function defined on $[0, 1] \to R$,

$f(x) = \dfrac{\sin x - x\cos x}{x^2}$, if $x \ne 0$ and f(0) = 0

Q.8 The function f(x)
(A) has a removable discontinuity at x = 0
(B) has a non removable finite discontinuity at x = 0
(C) has a non removable infinite discontinuity at x = 0
(D) Is continuous at x = 0

Q.9 $\int_0^1 f(x)\,dx$ equals
(A) 1 – sin1 (B) sin1 – 1
(C) sin1 (D) – sin1

Q.10 $\lim_{t\to 0} \dfrac{1}{t^2}\int_0^t f(x)\,dx$ equals
(A) $\frac{1}{3}$ (B) $\frac{1}{6}$ (C) $\frac{1}{12}$ (D) $\frac{1}{24}$

MATHEMATICS IIT JEE (JUNE 5th WEEK CLASS TEST 1) (FUNCTIONS) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○
										10	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	A,C	A	A,C	C	A	D	B	D	A	B

SOLUTIONS

Sol.1 (A, C)

As we know by mean value theorem

$$f'(c) = \frac{f(b) - f(a)}{b - a}, \ c \in (a, b)$$

$$g'(c) = \frac{g(b) - g(a)}{b - a}, \ c \in (a, b)$$

Put these values of f'(c) and g'(c) on right hand side which will become equal to left hand side.

Sol.2 (A)

Let f(x) = y, then we have

$$\begin{cases} y^3 - 5y^2 + 10y - 12 \le 0 \\ y^2 - 4y + 3 \ge 0 \\ y^2 - 6y + 8 \le 0 \end{cases}$$

On solving these inequalities, we get after common values as $y \in \{3\}$

$\Rightarrow$ f(x) = 3

(a constant and continous function)

Given +ve nos, b_1, b_2, b_3 are in G.P., then $\ell n\ b_1, \ell n\ b_2, \ell n\ b_3$, are in A.P.

$\Rightarrow$ $3 + \ell n\ b_1, 3 + \ell n\ b_2, 3 + \ell n\ b_3$ are in A.P.

$\Rightarrow$ $f(1) + \ell n\ b_1, f(2) + \ell n\ b_2, f(3) + \ell n\ b_3$ are in A.P.

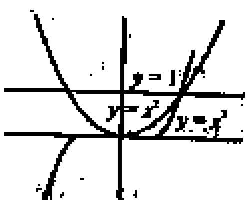
Sol.3 (A, C)

$f(x) = \min. \{1, x^2, x^3\}$

$$\Rightarrow f(x) = \begin{cases} x^3, & x \le 1 \\ 1, & x > 1 \end{cases}$$

$\Rightarrow$ $f(x) > 0, \forall x \in R$

and non-differentiable at x = 1.



Sol.4 (C)

$x = \pi$ (in general $n\pi$, $n \in I$)

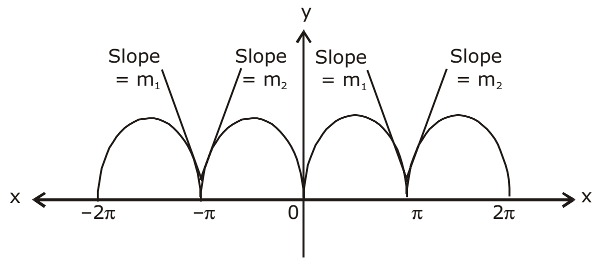
f(x) = |sinx| is continuous function

But RHD = m_2, LHD = m_1

$\therefore$ y = |sin x| is continuous function

Finally we can say that f(x) is continuous but non differentiable at number of points which can be counted if internal for x is known.

From the graph we note that f(x) is non differentiable at $x = 0, -\pi, \pi, 2\pi, -2\pi$ for $x \in [-2\pi, 2\pi]$



Sol.5 (A)

$$f(x) = \begin{cases} x^m \sin\left(\frac{1}{x}\right), & x \ne 0 \\ 0, & x = 0 \end{cases},$$

Hence,

$$f'(x) = mx^{m-1} \sin\left(\frac{1}{x}\right) + x^{m-2} \cos\left(\frac{1}{x}\right), x \ne 0$$

$$f'(0 + h) = \lim_{h \to 0} \frac{(0+h)^m \sin\left(\frac{1}{0+h}\right) - 0}{h} = 0$$

for m > 0

$$f'(0 - h) = \lim_{h \to 0} \frac{(0-h)^m \sin\left(\frac{1}{0-h}\right) - 0}{(-h)} = 0$$

for m > 0

and $m = \frac{p}{2q-1}$ where $p, q \in N$.

$\therefore$ f(x) is differentiable at x = 0 only when m > 0 and $m = \frac{p}{2q-1}$ type where $p, q \in N$.

Sol.6 (D)

$$\underset{x\to1-}{\text{Lt}} f(x) = \underset{x\to1-}{\text{Lt}} px^2 - q = p - q$$

$$\underset{x\to1+}{\text{Lt}} f(x) = \underset{x\to1}{\text{Lt}} (x+1) = 1 + 1 = 2$$

since f(x) is not continuous at x = 1

$\therefore \quad \underset{x\to1}{\text{Lt}} f(x) \neq f(1)$

$\therefore \quad p - q \neq 2$

$\therefore \quad p = 1, q = 1$

i.e. (p, q) = (1, 1)

[For other three values p – q = 2]

Sol.7 (B)

Since f(x) is continuous in [1, 3]

$\therefore$ it takes all values between f(1) and f(3) and it takes rational values for all x.

since 2 lies in the nbd. of the point 1.5 and f(2) = 10.

$\therefore \quad f(1.5) = 10$

Sol.8 (D)

$$\lim_{x\to0} f(x) = \lim_{x\to0} \frac{\cos x(\tan x - x)}{x^2}$$

$$f(0) = \lim_{x\to0} \frac{\sec^2 x - 1}{2x}$$

$$\Rightarrow \quad f(0) = \lim_{x\to0} \frac{\tan^2 x}{2x}$$

$\Rightarrow \quad f(0) = 0$

$\Rightarrow$ f(x) is continous.

Sol.9 (A)

$$\int_0^1 \frac{\sin x}{x^2} dx - \int_0^1 \frac{\cos x}{x} dx$$

$$= \left[\sin x\left(-\frac{1}{x}\right)\right]_0^1 + \int_0^1 \frac{\cos x}{x} dx - \int_0^1 \frac{\cos x}{x} dx$$

$$= \left[-\frac{\sin x}{x}\right]_0^1$$

$$= -\sin 1 + \lim_{x\to0} \frac{\sin x}{x}$$

$$= 1 - \sin 1$$

Sol.10 (B)

$$\lim_{t\to0} \frac{\int_0^t f(x)\,dx}{t^2} = \lim_{t\to0} \frac{\int_0^t \left(\frac{\sin x - x\cos x}{x^2}\right) dx}{t^2}$$

$$= \lim_{t\to0} \frac{\sin t - \cos t}{2t^3}$$

$$= \lim_{t\to0} \frac{\cos t(\tan t - t)}{2t^3}$$

$$= \lim_{t\to0} \frac{\sec^2 t - 1}{6t^2}$$

$$= \frac{1}{6} \lim_{t\to0} \frac{\tan^2 t}{t^2} = \frac{1}{6}$$