Dear student following is a Moderate level [○ ○ ● ○ ○] test paper. Score of 18 Marks in 10 Minutes would be a satisfactory performance. Questions 1-10(+3,–1) (Questions may have more than one correct option)

Q.1 Let $f(x) = \min\{\tan x, \cot x\}, \forall x \in R$, then points of discontinuity of f(x) are

(A) $\frac{n\pi}{2}, n \in I$ (B) $n\pi, n \in I$

(C) $\frac{n\pi}{3}, n \in I$ (D) None of these

Q.2 If f(x) be continuous function for all real values of x and satisfies; $x^2 + \{f(x) - 2\}x + 2\sqrt{3} - 3 - \sqrt{3}\,.\,f(x) = 0, \forall x \in R$. Then find the value of $f(\sqrt{3})$

(A) $2(1 - \sqrt{3})$ (B) $2(1 + \sqrt{3})$

(C) $2 - \sqrt{3}$ (D) None of these

Q.3 $f(x) = \begin{cases} \left(\tan\left(\frac{\pi}{4} + x\right)\right)^{1/x} & , \quad x \neq 0 \\ k & , \quad x = 0 \end{cases}$. For what value of k, f(x) is continuous at x = 0 ?

(A) e (B) e^2 (C) $2e^3$ (D) None

Q.4 Let $f(x) = \begin{cases} \dfrac{x^4 - 5x^2 + 4}{|(x-1)(x-2)|} & , \quad x \neq 1, 2 \\ = 6 & , \quad x = 1 \\ = 12 & , \quad x = 2 \end{cases}$.

Then f(x) is continuous on the set

(A) R (B) R – {1}

(C) R – {2} (D) R – {1, 2}

Q.5 The number of points at which the function $f(x) = \frac{1}{\log|x|}$ is discontinuous is

(A) 1 (B) 2 (C) 3 (D) 4

Q.6 If the function

$f(x) = \frac{x^2 - (A+2)x + A}{x - 2}$, for $x \neq 2$

$= 2$, for x = 2

is continuous at x = 2, then A is

(A) 0 (B) 1 (C) –1 (D) None

Q.7 Let f''(x) be continuous at x = 0 and f''(0) = 4. Then value of $\lim_{x\to 0} \frac{2f(x) - 3f(2x) + f(4x)}{x^2}$ is

(A) 11 (B) 2 (C) 12 (D) None

Q.8 Function $f(x) = (\sin 2x)^{\tan^2 2x}$ is not defined at $x = \frac{\pi}{4}$. If f(x) is continuous at $x = \frac{\pi}{4}$ then $f\left(\frac{\pi}{4}\right)$ is equal to

(A) 1 (B) 2 (C) $\sqrt{e}$ (D) None

Q.9 The value of f(0) so that the functions $f(x) = \frac{2x - \sin^{-1}x}{2x + \tan^{-1}x}$ is continuous at each point on its domain is

(A) 2 (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $-\frac{1}{3}$

Q.10 Given the function $f(x) = \frac{1}{(1-x)}$. The points of discontinuity of the composite function, y = f(f(f(x))) are at x =

(A) 0 (B) 1 (C) 2 (D) –1

MATHEMATICS IIT JEE (JUNE 5th WEEK CLASS TEST 2) (CONTINIUITY & DIFFERENTIABILITY) ANSWER KEY

Name : .... Roll No. : ....

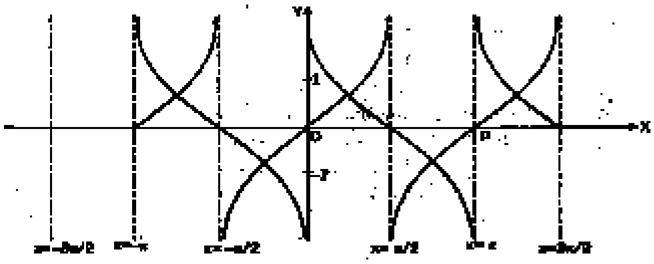
	A	B	C	D
1	○	○	○	○
2	○	○	○	○
3	○	○	○	○
4	○	○	○	○
5	○	○	○	○
6	○	○	○	○
7	○	○	○	○
8	○	○	○	○
9	○	○	○	○
10	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	C	A	B	D	C	A	C	C	B	A,B

SOLUTIONS

Sol.1 (C)



Thus the points of discontiniuty are $\pm\pi$, $\pm\frac{\pi}{2}$, 0, ... which can be put in form of $\frac{n\pi}{2}$, $n \in I$.

Sol.2 (A)

As f(x) is continuous for $x \in R$.

Thus, $\lim\limits_{x\to\sqrt{3}} f(x) = f(\sqrt{3})$

where $f(x) = \dfrac{x^2 - 2x + 2\sqrt{3} - 3}{\sqrt{3} - x}$, $x \neq \sqrt{3}$

$$\lim_{x\to\sqrt{3}} f(x) = \lim_{x\to\sqrt{3}} \frac{x^2 - 2x + 2\sqrt{3} - 3}{\sqrt{3} - x}$$

$$= \lim_{x\to\sqrt{3}} \frac{(2 - \sqrt{3} - x)(\sqrt{3} - x)}{(\sqrt{3} - x)}$$

$$= 2(1 - \sqrt{3})$$

$\therefore$ $f(\sqrt{3}) = 2(1 - \sqrt{3})$

Sol.3 (B)

Here, $\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0} \left\{\tan\left(\frac{\pi}{4} + x\right)\right\}^{1/x}$

$\Rightarrow \lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0} \left[\frac{1 + \tan x}{1 - \tan x}\right]^{1/x}$ (1^∞ form)

$\Rightarrow \lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0} \left[1 + \left(\frac{1 + \tan x}{1 - \tan x} - 1\right)\right]^{1/x}$

$\Rightarrow \lim\limits_{x\to 0} f(x) = e^{\lim\limits_{x\to 0}\left(\frac{2\tan x}{1-\tan x}\right)\frac{1}{x}}$

$\Rightarrow \lim\limits_{x\to 0} f(x) = e^{2\lim\limits_{x\to 0}\frac{\tan x}{x(1-\tan x)}} = e^2$

Here f(x) is continuous at x = 0 when,

$\lim\limits_{x\to 0} f(x) = f(0)$

$\Rightarrow$ $k = e^2$

Sol.4 (D)

$$f(x) = \frac{(x^2 - 1)(x^2 - 4)}{|(x - 1)(x - 2)|}$$

Consider continuity at x = 1. Let us redefine the function for x < 1, 1 < x < 2

$x < 1, f(x) = \dfrac{(x^2 - 1)(x^2 - 4)}{(x - 1)(x - 2)} = (x + 1)(x + 2)$

$x > 1, f(x) = \dfrac{(x^2 - 1)(x^2 - 4)}{-(x - 1)(x - 2)} = -(x + 1)(x + 2)$

L.H.L. $= \mathop{Lt}\limits_{h\to 0} (1 - h + 1)(1 - h + 2)$

$= \mathop{Lt}\limits_{h\to 0} (2 - h)(3 - h) = 6$

R.H.L. $= \mathop{Lt}\limits_{h\to 0} -[1 + h + 1][1 + h + 2]$

$= \mathop{Lt}\limits_{h\to 0} -(2 + h)(3 + h) = -6$

At x = 1, value = 0.

Since L $\neq$ R therefore f(x) is not continuous at x = 1. Similarly it can be shown that f(x) is not continuous at x = 2.

Hence it is continuous on the set R – {1, 2}

Sol.5 (C)

f(x) is not defined for log |x| = 0 or |x| = 1 or x = +1, –1. Again by definition of log x, f(x) is not defined for |x| = 0 or x = 0. Thus we have in all three points of discontinuity.

Sol.6 (A)

For continuity, lim = value = 2 (given)

$$\lim_{x\to 2} f(x) = \frac{0}{0}. \text{ Apply L' Hospital's rule}$$

$\therefore$ $\lim = \lim_{x\to 2} \{2x - (A+2)\} = 2 - A$

$\therefore$ 2 – A = 2

$\Rightarrow$ A = 0

Sol.7 (C)

If f"(x) is continuous at x = 0, then

lim f"(0) = f"(0) i.e. Lt = value

or lim f"(0) = 4 .....(1)

$$\text{Now } \lim_{x\to 0} \phi(x) = \lim_{x\to 0} \frac{2f(x) - 3f(2x) + f(4x)}{x^2} \left(\frac{0}{0}\right)$$

for x = 0

$$= \lim_{x\to 0} \frac{2f'(x) - 6f'(2x) + 4f'(4x)}{2x} \left(\frac{0}{0}\right)$$

L' Hospital

$$= \lim_{x\to 0} \frac{2f''(x) - 12f''(2x) + 16f''(4x)}{2}$$

= (1 – 6 + 8) lim f"(0) = 3.4 = 12

Sol.8 (C)

For continuity we know that Limit = Value

$$\therefore \quad f\left(\frac{\pi}{4}\right) = \lim_{x\to \pi/4} (\sin 2x)^{\tan^2 2x} \quad [\text{form } (1^\infty)]$$

$$= \lim_{x\to \pi/4} (\sin^2 2x)^{\frac{1}{2}\tan^2 2x}$$

$$= \lim_{x\to \pi/4} (1 - \cos^2 2x)^{\frac{1}{2}\tan^2 2x}$$

$$= \lim_{x\to \pi/4} \left[(1 - \cos^2 2x)^{1/\cos^2 2x}\right]^{\frac{1}{2}\sin^2 2x}$$

$$= e^{1/2\,.\,1} = \sqrt{e}$$

Sol.9 (B)

For continuity, lim = value

$$f(x) = \lim_{x\to 0} \frac{2 - \frac{\sin^{-1} x}{x}}{2 + \frac{\tan^{-1} x}{x}} = \frac{2-1}{2+1} = \frac{1}{3}$$

By $x = \sin\theta$, $\underset{\theta\to 0}{\text{Lt}} \dfrac{\theta}{\sin\theta} = 1.$

Similarly, $\underset{\theta\to 0}{\text{Lt}} \dfrac{\theta}{\tan\theta} = 1.$

Sol.10 (A, B)

The point x = 1 is clearly a point of discontinutiy of the function

$$y = f(x) = \frac{1}{1-x}$$

If $x \neq 1$, then

$$v(x) = f[f(x)] = f\left(\frac{1}{1-x}\right)$$

$$= \frac{1}{1 - [1/(1-x)]} = \frac{x-1}{x}$$

Hence, the point x = 0 is a point of discontinuity of the function v.

If $x \neq 0$, $x \neq 1$, then

w(x) = f[f{f(x)}]

$$= f\left[f\left(\frac{1}{1-x}\right)\right] = f\left(\frac{x-1}{x}\right)$$

$$= \frac{1}{1-(x-1)/x} = x.$$

Hence w is clearly continuous everywhere, Thus, the points of discontinuity of the composite function f[f{f(x)}] are x = 0, x = 1.