Dear student following is a Moderate level [○ ○ ● ○ ○] test paper. Score of 18 Marks in 15 Minutes would be a satisfactory performance. Questions 1-9(+3,–1) (Questions may have more than one correct option)

Q.1 Number of points of discontinuity of y = g(u) $= \frac{1}{u^2 + u - 2}$, where $u = \frac{1}{x-1}$ is/are -

(A) 3 (B) 1 (C) 2 (D) None

Q.2 If $f(x) = \frac{1}{(x-1)(x-2)}$ and $g(x) = \frac{1}{x^2}$. Then points of discontinuity of f(g(x)) are

(A) $\left\{-1, 0, 1, \frac{1}{\sqrt{2}}\right\}$ (B) $\left\{-\frac{1}{\sqrt{2}}, -1, 0, 1, \frac{1}{\sqrt{2}}\right\}$

(C) {0, 1} (D) $\left\{0, 1, \frac{1}{\sqrt{2}}\right\}$

Q.3 If f(x) is continuous in [0, 2] and f(0) = f(2). Then the equation f(x) = f(x + 1) has

(A) Non real root in [0, 2]
(B) At least one real root in [0, 1]
(C) At lest one real root in [0, 2]
(D) At least one real root in [1, 2]

Q.4 The function f(x) = 1 + |sin x| is

(A) continuous nowhere
(B) continuous everywhere
(C) not differentiable at x = 0
(D) not differentiable at an infinite number of points

Q.5 The function f(x) = a[1 + x] + b[x – 1], where [x] is the greatest integer function is continuous at x = 1, if

(A) a + b = 0 (B) a = b
(C) 2a – b = 0 (D) None

Q.6 Let f(x) be defined for all x > 0 and be continuous. Let f(x) satisfy f(x/y) = f(x) – f(y) for all x, y and f(e) = 1. Then

(A) $f(1/2) \to 0$ as $x \to \infty$
(B) $x f(x) \to 0$ as $x \to 0$
(C) f(x) is unbounded
(D) f(x) = log x.

Q.7 If function $f(x) = \max\{x^2, (x-1)^2, 2x(1-x)\}$, $0 \le x \le 1$. Then which of the following is true

(A) f(x) is differentiable for all x
(B) f(x) is differentiable for all x expect at one point
(C) f(x) is differentiable for all x expect at two points
(D) f(x) is not differentiable at more than two points

Q.8 f(x) and g(x) are continuous in [0, 1] and differentiable in (0, 1) such that f(0) = 2, g(0) = 0, f(1) = 6, g(1) = 2. Then in the interval (0, 1) f'(x) = 2g'(x) holds for

(A) At least one value of x
(B) At most one value of x
(C) more than one value of x
(D) None of these

Q.9 Match List I with List II

List I	List II
I. $f(x) = (1 + 3x)^{1/x}$, $(x \neq 0)$, $f(0) = e^3$	(a) Discontinuity of first find at x = 0
II. $f(x) = \frac{\sqrt{1+x} - 1}{x}$, $(x \neq 0)$, $f(0) = 1$	(b) Discontinuity of second find at x = 0
III. $f(x) = x + [x]$, $x \in Z$	(c) Removable discontinuity at x = 0
IV. f(x) = log x	(d) Continuity at x = 0

(A) I - d, II - c, III - b, IV - a
(B) I - d, II - c, III - a, IV - b
(C) I - c, II - d, III - a, IV - b
(D) I - c, II - d, III - b, IV - a

MATHEMATICS IIT JEE (JUNE 5th WEEK CLASS TEST 3) (CONTINIUITY & DIFFERENTIABILITY) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9
Ans.	A	B	B	B,C,D	A	B,C,D	C	A	B

SOLUTIONS

Sol.1 (A)

The function $u = f(x) = \frac{1}{x-1}$ is discontinuous at the point x = 1.

The function

$$y = g(u) = \frac{1}{u^2 + u - 2} = \frac{1}{(u+2)(u-1)}$$

is discontinous at u = 1, −2

$\therefore$ when $u = 1 \Rightarrow \frac{1}{x-1} = 1 \Rightarrow x = 2$

$\therefore$ when $u = -2 \Rightarrow \frac{1}{x-1} = -2 \Rightarrow x = \frac{1}{2}$

Hence the composite function y = g(f(x)) is discontinuous at three point $\left\{1, 2, \frac{1}{2}\right\}$.

Sol.2 (B)

$$f(g(x)) = f\left(\frac{1}{x^2}\right) = \frac{1}{\left(\frac{1}{x^2} - 1\right)\left(\frac{1}{x^2} - 2\right)}$$

$$= \frac{x^4}{(1 - x^2)(1 - 2x^2)}$$

$\Rightarrow$ f(g(x)) is discontinous at $x = \pm 1, x = \pm \frac{1}{\sqrt{2}}$

and also at 0, since g(x) is discontinuous at x = 0.

Sol.3 (B)

Let $\phi(x) = f(x) - f(x + 1)$

$\Rightarrow \phi(0) = f(0) - f(1)$

and $\phi(1) = f(1) - f(2)$

$\Rightarrow \phi(0) + \phi(1) = 0$

$\Rightarrow \phi(0)$ and $\phi(1)$ are of opposite sign.

$\Rightarrow$ f(x) = f(x + 1) has at least one real root in [0, 1]

Sol.4 (B, C, D)

The function is clearly continuous everywhere i.e. (B). For if a is any real number, then

$f(a) = 1 + |\sin a|$

$f(a + 0) = \lim_{h\to 0}[1+|\sin(a+h)|]$

$= 1 + |\sin a|$

and $f(a - 0) = \lim_{h\to 0}[1+|\sin(a-h)|]$

$= 1 + |\sin a|$

The function is not differentiable at $x = n\pi$. n = 0, ±1, ±2, ... as shown below: i.e. (c) and (d).

$f(n\pi) = 1 + |\sin n\pi| = 1 + |0| = 1$

Now $Rf'(n\pi) = \lim_{h\to 0}\frac{1+|\sin(n\pi+h)|-1}{h}$ (h > 0)

$= \lim_{h\to 0}\frac{|\pm\sinh|}{h} = \lim_{h\to 0}\frac{\sinh}{h} = 1$

[$\because$ h is small and > 0 and as such sin h > 0] = 1

and $Lf'(n\pi) = \lim_{h\to 0}\frac{1+|\sin(n\pi-h)|-1}{-h}$

$= \lim_{h\to 0}\frac{|\pm\sinh|}{-h} = \lim_{h\to 0}\frac{\sinh}{-h} = -1$

Since $Rf'(n\pi) \neq Lf'(n\pi)$, f(x) is not differentiable at $x = n\pi$, n = 0, ±1, ±2, ...

At all other point f(x) is clearly differentiable. For if a is any real number such that

$n\pi < a < (n + 1)\pi$.

Sol.5 (A)

Since for f(x) to be continous at any point, L.H.L. should be equal to R.H.L.

$\therefore \lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$

$\Rightarrow \lim_{h\to 0} a[1 + h + 1] + b[1 + h - 1]$

$= \lim_{h\to 0} a[1 - h + 1] + b[1 - h - 1]$

$\Rightarrow \lim_{h\to 0} a[2 + h] + b[h] = \lim_{h\to 0} a[2 - h] + b[-h]$

$\Rightarrow a(2) + b(0) = a(1) + b(-1)$

$\Rightarrow 2a - a + b = 0$

$\Rightarrow a + b = 0$

Sol.6 (B, C, D)

Clearly f(x) = log x

$[\because \log\frac{x}{y} = \log x - \log y$ and $\log e = 1]$

$\therefore$ (D) holds.

Since $\underset{x\to 0}{\text{Lt}}\ x\log x = 0$

$\therefore$ $\underset{x\to 0}{\text{Lt}}\ x f(x) = 0$

$\therefore$ (B) holds.

$f(x) = \log \to \infty$ as $x \to \infty$

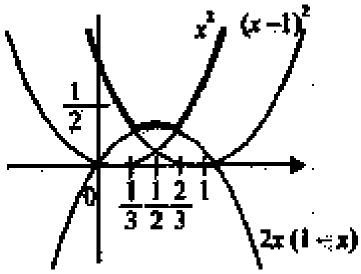
$\therefore$ f(x) is unbounded

$\therefore$ (C) holds.

$f\left(\frac{1}{2}\right) = \log\frac{1}{2} = -\log 2 \neq 0$

$\therefore$ (A) does not hold.

Sol.7 (C)



Graph of f(x) is shown by dark lines. Graph of f(x) has two sharp points at $x=\frac{1}{3}$ and $\frac{2}{3}$.

$\Rightarrow$ f(x) isn't differentiable at two points.

Sol.8 (A)

Apply cauchy's mean value theorem

$$\frac{f'(x)}{g'(x)} = \frac{f(b)-f(a)}{g(b)-g(a)}$$

where $x \in (a, b)$; substituting the value

$$\frac{f'(x)}{g'(x)} = \frac{6-2}{2-0} \text{ for } x \in (0, 1)$$

$\therefore$ $f'(x) = 2g'(x)$ for $x \in (0, 1)$

For At least one value of $x \in (0, 1)$.

Sol.9 (B)

I. $\underset{x\to 0}{\text{Lt}}\ f(x) = \underset{x\to 0}{\text{Lt}}(1+3x)^{1/x}$

$$= \underset{x\to 0}{\text{Lt}}\left((1+3x)^{\frac{1}{3x}}\right)^3$$

$= e^3 = f(0)$

$\therefore$ f is continuous at x = 0.

$\therefore$ I $\leftrightarrow$ d

II. $\underset{x\to 0}{\text{Lt}}\ f(x) = \underset{x\to 0}{\text{Lt}}\ \frac{\sqrt{1+x}-1}{x}\cdot\frac{\sqrt{1+x}+1}{\sqrt{1+x}+1}$

$$= \underset{x\to 0}{\text{Lt}}\ \frac{1+x-1}{x(\sqrt{1+x}+1)} = \frac{1}{2} \neq f(0) = 1$$

$= e^3 = f(0)$

$\therefore$ f is discontinuous at x = 0.

This is removable discontinuity II $\leftrightarrow$ c

III. $\underset{x\to 0-}{\text{Lt}}\ f(x) = \underset{x\to 0-}{\text{Lt}}\ x + (x)$

$= 0 + [0 -] = -1$

$\underset{x\to 0+}{\text{Lt}}\ f(x) = \underset{x\to 0+}{\text{Lt}}\ x + [x]$

$= 0 + [0 +] = 0$

$\therefore$ $\underset{x\to 0}{\text{Lt}}\ f(x)$ does not exist.

$\therefore$ f(x) is discontinuous at x = 0.

This is discontinuity of the first kind

$\therefore$ III $\leftrightarrow$ a

IV. $\underset{x\to 0}{\text{Lt}}\ f(x) = \underset{x\to 0}{\text{Lt}}\ \log x = -\infty$

$\therefore$ f is discontinuous at x = 0.

This is discontinuity of the second kind

$\therefore$ IV $\leftrightarrow$ b.