Dear student following is a Moderate level [○ ○ ● ○ ○] test paper. Score of 18 Marks in 15 Minutes would be a satisfactory performance. Questions 1-10(+3,–1) (Questions may have more than one correct option)

Q.1 If $f(x) = \frac{x-1}{2x^2-7x+5}$, when $x \neq 1$

$= -\frac{1}{3}$, when x = 1, then f'(1) =

(A) $\frac{2}{9}$ (B) $\frac{9}{2}$ (C) $-\frac{2}{9}$ (D) $\frac{1}{9}$

Q.2 A function f is defined s.t for all x and y f(x + y) = f(x) + f(y) and f(x) = 1 + xg(x), where $\lim_{x\to 0} g(x) = 1$, then f'(x) =

(A) $\frac{1}{2}f(x)$ (B) 2f(x)
(C) f(x) (D) $-\frac{1}{2}f(x)$

Q.3 Let f(x) = min {x, x^2}, ∀ ∈ R, then no. of points for which f(x) is not differentiable are

(A) 1 (B) 2
(C) infinite (D) None

Q.4 The number of points in (1, 3), where $f(x) = a^{[x^2]}$, a > 1 is not differentiable where [x] denotes the integral part of x.

(A) 0 (B) 7
(C) 2 (D) 5

Q.5 Value of a such that

$$f(x) = \frac{(e^x-1)^3 \operatorname{cosec} ax}{\ell n(1+x^2)}, x \neq 0$$

$$= b \qquad , x = 0$$

is continuous at x = 0 is equal to

(A) b (B) –1/b (C) –b (D) 1/b

Q.6 $f(x) = (x^2 - 4)\,|x^2 - 5x + 6| + |\cos|x||$ is non differentiable at

(A) x = 0 (B) x = 2 (C) x = –2 (D) x = 3

Q.7 If the function $f(x) = \left[\frac{(x-2)^3}{a}\right] \sin(x-2) +$ a cos (x – 2), (where [.] denotes the greatest integer function) is continuous and differentiable in (4, 6), then

(A) a ∈ [8, 64] (B) a ∈ (0, 8]
(C) a ∈ [64, ∞) (D) None of these

Q.8 If f(x) is differentiable and (f(x).g(x)) is differentiable at x = a, then

(A) g(x) must be differentiable at x = a
(B) If g(x) is discontinuous, then f(a) = 0
(C) f(a) ≠ 0, then g(x) must be differentiable
(D) None of these

Passage :

f : R → R such that f(x) = sgn (x) whose range is set of number {–1, 0, 1} for different domain of f where sgn (x) represent signum function

Q.9 The function $\sqrt{1+f(x)}$ has / is

(A) Removable discontinuity
(B) Discontinuous at x = 0
(C) Continuous at x = 0
(D) None of these

Q.10 The function g(x) = 2 + |f(x)| has

(A) Infinite discontinuity
(B) Non-discontinuous at x = 0
(C) Removable discontinuity
(D) Jump discontinuty

MATHEMATICS IIT JEE (JUNE 5th WEEK CLASS TEST 4) (CONTINIUITY & DIFFERENTIABILITY) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○
										10	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	C	C	B	C	D	D	C	B,C	B	C

SOLUTIONS

Sol.1 (C)

$\because \quad f(x) = \dfrac{x-1}{2x^2 - 7x + 5}$ when $x \neq 1$

$\therefore \quad f(1+h) = \dfrac{1+h-1}{2(1+h)^2 - 7(1+h) + 5}$

$$= \frac{h}{2h^2 - 3h} = \frac{1}{2h-3}$$

From definition

$$f'(1) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{2h-3} - \left(-\frac{1}{3}\right)}{h}$$

$$= \lim_{h\to 0} \frac{3 + 2h - 3}{h(2h-3)3}$$

$$= \lim_{h\to 0} \frac{2h}{3h(2h-3)}$$

$$= \lim_{h\to 0} \frac{2}{3(2h-3)}$$

$$= -\frac{2}{9}$$

Sol.2 (C)

$$\because \quad f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x+0)}{h}$$

$$= \lim_{h\to 0} \frac{f(x)f(h) - f(x)f(0)}{h}$$

$$= f(x) \lim_{h\to 0} \frac{f(h) - f(0)}{h}$$

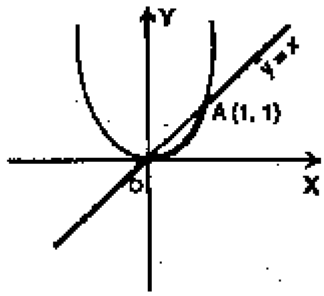
$$= f(x) \lim_{h\to 0} \frac{\{1 + hg(h)\} - \{1 + 0.g(0)\}}{h}$$

$$= f(x) \lim_{h\to 0} g(h)$$

$$= f(x) \lim_{x\to 0} g(x) \quad \left(\because \lim_{x\to 0} g(x) = 1\right)$$

$$= f(x)$$

Sol.3 (B)



$f(x) = \min. \{x, x^2\}$

graph of f(x) has been shown in the figure.

From graph it is clear that f(x) is continuous at all x but not differentiable at x = 0 and x = 1.

Sol.4 (C)

Here $1 < x < 3$ and in this interval x^2 is an increasing function and $1 < x^2 < 9$.

$\therefore \quad [x^2] = 1, \quad 1 \le x \le \sqrt{2}$

$= 2, \quad \sqrt{2} \le x \le \sqrt{3}$

$= 3, \quad \sqrt{3} \le x < 2$

$= 4, \quad 2 \le x < \sqrt{5}$

$= 5, \quad \sqrt{5} \le x < \sqrt{6}$

$= 6, \quad \sqrt{6} \le x < \sqrt{7}$

$= 7, \quad \sqrt{7} \le x < \sqrt{8}$

$= 8, \quad \sqrt{8} \le x < 3$

Clearly $[x^2]$ and also $a^{[x^2]}$ is discontinuous and not differentiable at only 7 points.

Sol.5 (D)

Given f(x) is continuous at x = 0

$$\Rightarrow \lim_{x\to 0^+} f(x) = f(0)$$

$$\Rightarrow \lim_{h\to 0} \left\{\frac{(e^h - 1)^3 \operatorname{cosec} ah}{\log(1+h^2)}\right\} = b$$

$$\Rightarrow \lim_{h\to 0} \left\{\frac{(e^h - 1)^3}{\log(1+h^2)\sin ah}\right\} = b$$

$$\Rightarrow \lim_{h\to 0}\left\{\frac{h^3(e^h-1)^3}{\frac{h^3h^2}{h^2ah}ahlog(1+h^2)\,sinah}\right\} = b$$

$$\Rightarrow \frac{1}{a} = b \qquad \Rightarrow \qquad a = \frac{1}{b}$$

Sol.6 (D)

$\because$ cos |x| = cos x, so the doubtful points are x = 2, 3

$f(x) = (x^2 - 4)(x^2 - 5x + 6) + \cos x, \quad x < 2, x > 3$

$= -(x^2 - 4)(x^2 - 5x + 6) + \cos x, \quad 2 \le x \le 3$

$f'(x) = 2x(x^2 - 5x + 6) + (x^2 - 4)(2x - 5) - \sin x, \quad x < 2, x > 3$

$= -2x(x^2 - 5x + 6) - (x^2 - 4)(2x - 5) - \sin x, \quad 2 \le x \le 3$

$\therefore$ f'(2 – 0) = – sin2 and f'(2 + 0) = – sin2

and f'(3 – 0) = – 5 – sin3, f'(3 + 0) = 5 – sin3

Hence f(x) is not differentiable at x = 3.

Sol.7 (C)

We have $x \in (4, 6)$

$\Rightarrow 2 < x - 2 < 4$

$$\Rightarrow \frac{8}{a} < \frac{(x-2)^3}{a}, \frac{64}{a}, \qquad a > 0$$

For f(x) to be continuous and differentiable in (4, 6), $\left[\frac{(x-2)^3}{a}\right]$ must attain a constant value for $x \in (4, 6)$

Clearly, this is possible only when $a \ge 64$

In that case, we have

f(x) = a cos (x – 2) which is continuous and differentiable

$\therefore a \in [64, \infty)$

Sol.8 (B, C)

$$\left[\frac{d}{dx}(f(x).g(x))\right]_{x=a}$$

$$= f'(a)g(a) + \lim_{h\to 0}\frac{g(a+h)-g(a)}{h}.f(a)$$

If $f(a) \neq 0$

$\Rightarrow$ g'(a) must exist.

Also if g(a) is discontinuous, f(a) must be 0 for f(x).g(x) to be differentiable.

Sol.9 (B)

$\sqrt{1+f(x)} = \sqrt{1+sgn(x)}$ is defined $\forall \in R$

but sgn (x) is discontinious at x = 0

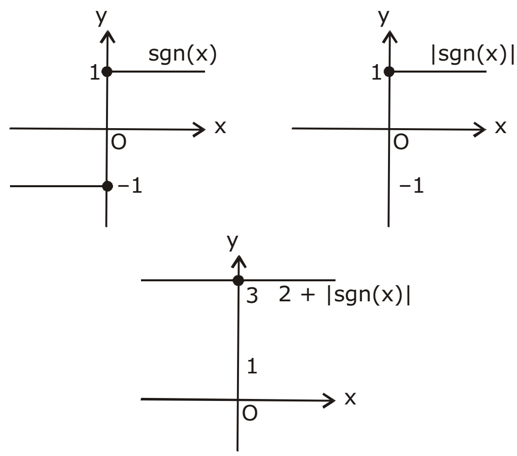
therefore $\sqrt{1+sgn(x)}$ is also discontinuous at x = 0.

more over $\underset{x\to 0^-}{LHL} \neq \underset{x\to 0^+}{RHL}$ i.e. non removable discontiniuity.

Sol.10 (C)

At x = 0

LHL = RHL = 3 $\neq$ g(0) = 2



g(x) = 2 + |f(x)|