Dear student following is an Easy level [● O O] test paper. Score of 24 Marks in 15 Minutes would be a satisfactory performance. Questions 1-10 (+3, –1). (All questions have only one option correct)

Q.1 If $I = \int_0^{\pi/2} \frac{\sin 8x \log(\cot x)}{\cos 2x} dx$, then I equals-

(A) $-\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $-\frac{1}{3}$ (D) 0

Q.2 If $I = \int_{1/e}^{e} |\log x| \frac{dx}{x^2}$, then I equals-

(A) 2 (B) 2/e
(C) 2(1 – 1/e) (D) 0

Q.3 If b > a, and $I = \int_a^b \sqrt{\frac{x-a}{b-x}} dx$, then I equals-

(A) $\frac{\pi}{2}(b - a)$ (B) $\pi(b - a)$

(C) $\frac{\pi}{2}$ (D) $2\pi(b - a)$

Q.4 If $I = \int_0^{2\pi} e^{x/2} \sin\left(\frac{x}{2} + \frac{\pi}{4}\right) dx$, then I equals-

(A) π (B) 0 (C) $-\frac{\pi}{2}$ (D) 2π

Q.5 If $\int_0^1 \frac{\sin t}{1+t} dt = \alpha$, then value of $I = \int_{4\pi-2}^{4\pi} \frac{\sin(x/2)}{4\pi + 2 - x} dx$ is-

(A) $\frac{\alpha}{2}$ (B) $-\alpha$ (C) $-\frac{\alpha}{2}$ (D) α

Q.6 If $h(x) = \int_1^x \sin^4 t \, dt$, then $h(x + \pi)$ equals-

(A) $h(x) + h(\pi)$ (B) $h(x)\, h(\pi)$
(C) $h(x) - h(\pi)$ (D) $h(x)/h(\pi)$

Q.7 Let f(x) = x – [x], where for $x \in R$, [x] denotes the greatest integer $\le$ x. Then $I = \int_{-2}^{2} f(x)\, dx$ equals-

(A) –2 (B) –1 (C) 0 (D) 2

Q.8 If $0 < \alpha < 1$ and $I = \int_{-1}^{1} \frac{dx}{\sqrt{1 - 2\alpha x + \alpha^2}}$, then I equals-

(A) $\frac{1}{\alpha}$ (B) $\frac{2}{\alpha}$ (C) $\frac{3}{\alpha}$ (D) None

Q.9 If $I = \int_8^{15} \frac{dx}{(x-3)\sqrt{x+1}}$, then I equals-

(A) $\frac{1}{2} \log \frac{5}{3}$ (B) $2 \log \frac{1}{3}$

(C) $\frac{1}{2} \log \frac{1}{5}$ (D) $2 \log \frac{5}{3}$

Q.10 If $I = \int_0^1 x \sqrt{\frac{1-x}{1+x}} dx$, then I equals-

(A) $1 + \frac{\pi}{4}$ (B) $1 - \frac{\pi}{4}$

(C) π (D) $\pi - \sqrt{2}$

MATHEMATICS IIT JEE (OCT. 1st WEEK CLASS TEST 1) (DEFINITE INTEGRATION) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○
										10	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	D	C	A	B	B	A	D	D	A	B

SOLUTIONS

Sol.1 (D)

Using $\int_0^{\pi/2} f(x)\,dx = \int_0^{\pi/2} f(a-x)\,dx$,

$$I = \int_0^{\pi/2} \frac{\sin(4\pi - 8x)\log\cot(\pi/2 - x)}{\cos(\pi - 2x)}\,dx$$

$$= \int_0^{\pi/2} \frac{(-\sin 8x)(\log\tan x)}{-\cos 2x}\,dx$$

$= -I$ [$\because$ $\log\tan x = -\log\cot x$]

$\Rightarrow 2I = 0$ or $I = 0$

Sol.2 (C)

Put $\frac{1}{x} = t$, so that

$$I = \int_e^{1/e} \left|\log\frac{1}{t}\right| (-1)\,dt$$

$$= \int_{1/e}^{e} |-\log t|\,dt$$

$$= \int_{1/e}^{1} (-\log t)\,dt + \int_1^e (\log t)\,dt$$

$$= (-t\log t + t)]_{1/e}^{1} + (t\log t - t)]_1^e$$

$$= 1 - \frac{1}{e} - \frac{1}{e} + e - e + 1$$

$= 2(1 - 1/e)$

Sol.3 (A)

Put $b - x = t^2$, so that

$$I = \int_{\sqrt{b-a}}^{0} \sqrt{\frac{b - t^2 - a}{t^2}}\,(-2t)\,dt$$

$= 2\int_0^c \sqrt{c^2 - t^2}\,dt$ where $c = \sqrt{b-a}$

$$= 2\left[\frac{1}{2}t\sqrt{c^2 + t^2} + \frac{c^2}{2}\sin^{-1}\left(\frac{t}{c}\right)\right]_0^c$$

$= 0 + c^2\sin^{-1}(1) - 0$

$= \frac{\pi}{2}(b - a)$.

Sol.4 (B)

Put $\frac{x}{2} = \theta$, so that

$$I = 2\int_0^{\pi} e^{\theta} \sin\left(\theta + \frac{\pi}{4}\right)d\theta$$

$$= \sqrt{2}\int_0^{\pi} e^{\theta} (\sin\theta + \cos\theta)\,d\theta$$

$$= \sqrt{2}\, e^{\theta} \sin\theta]_0^{\pi} = 0.$$

[$\because \int e^x (f(x) + f'(x))\,dx = e^x f(x)$]

Sol.5 (B)

Put $\frac{x}{2} = \theta$, so that

$$I = \int_{2\pi-1}^{2\pi} \frac{\sin\theta}{4\pi + 2 - 2\theta}(2)\,d\theta$$

$$= \int_{2\pi-1}^{2\pi} \frac{\sin\theta}{2\pi + 1 - \theta}\,d\theta$$

Now, put $2\pi - \theta = t$, so that

$$I = \int_1^0 \frac{\sin(2\pi - t)}{1 + t}(-1)\,dt$$

$$= -\int_0^1 \frac{\sin t}{1 + t}\,dt = -\alpha$$

Sol.6 (A)

$h(x + \pi) + h(\pi) + I$

Where $I = \int_{\pi}^{x+\pi} \sin^4 t\,dt$

Put $t = \theta + \pi$, so that

$$I = \int_0^x (\sin(\pi + \theta))^4 \, d\theta$$

$$= \int_0^x (-\sin\theta)^4 \, d\theta = h(x)$$

$\therefore$ $h(x + \pi) = h(\pi) + h(x)$.

Sol.7 (D)

$$I = \int_{-2}^{2} (x - [x])\, dx = 0 - \int_{-2}^{2} [x]\, dx$$

$$= -\left[\int_{-2}^{-1}(-2)\,dx + \int_{-1}^{0}(-1)\,dx + \int_0^1 0\,dx + \int_1^2 dx\right]$$

$$= -(-2 - 1 + 0 + 1) = 2.$$

Sol.8 (D)

$$I = \int_{-1}^{1}(1 - 2\alpha x + \alpha^2)^{-1/2}\, dx$$

$$= \left.\frac{2(1 - 2\alpha x + \alpha^2)^{1/2}}{-2\alpha}\right]_{-1}^{1}$$

$$= -\frac{1}{\alpha}[(1 - 2\alpha + \alpha^2)^{1/2} - (1+2\alpha + \alpha^2)^{1/2}]$$

$$= -\frac{1}{\alpha}[(1 - \alpha) - (1 + \alpha)] = 2.$$

Sol.9 (A)

Put $\sqrt{x+1} = t$ or $x + 1 = t^2$

$$\therefore \quad I = \int_3^4 \frac{2t}{(t^2 - 4)t}\, dt = \frac{2}{(2)\,(2)} \log \left.\left|\frac{t-2}{t+2}\right|\right]_3^4$$

$$= \frac{1}{2}\left[\log\frac{1}{3} - \log\frac{1}{5}\right]$$

$$= \frac{1}{2} \log \frac{5}{3}$$

Sol.10 (B)

We can write

$$I = \int_0^1 \frac{x(1-x)}{\sqrt{1-x^2}}\, dx$$

$$= \int_0^1 \left(\frac{x}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}} + \frac{1-x^2}{\sqrt{1-x^2}}\right) dx$$

$$= \left.\left(-\sqrt{1-x^2} - \sin^{-1} x\right)\right]_0^1$$

$$+ \left.\left(\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1} x\right)\right]_0^1$$

$$= -\frac{\pi}{2} + 1 + \frac{1}{2}\left(\frac{\pi}{2}\right) = 1 - \frac{\pi}{4}.$$