Dear student following is a Moderate level [O ● O] test paper. Score of 18 Marks in 15 Minutes would be a satisfactory performance. Questions 1-10 (+3, –1). (All questions have only one option correct)

Q.1 If $S_n = \left[\frac{1}{1+\sqrt{n}} + \frac{1}{2+\sqrt{2n}} + \ldots\ldots + \frac{1}{n+\sqrt{n^2}}\right]$, then $\lim_{n\to\infty} S_n =$

(A) log 2 (B) log 4
(C) log 8 (D) None of these

Q.2 $\int_{-\pi/2}^{\pi/2} \sin|x|\, dx$ is equal to-

(A) 1 (B) 2 (C) –1 (D) –2

Q.3 If $I = \int_0^1 \cos\left(2\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right) dx$, then-

(A) $I = -\frac{1}{2}$ (B) $0 < I < \frac{1}{2}$

(C) $I > \frac{1}{2}$ (D) None of these

Q.4 The value of $\int_0^{\pi/2} \frac{\left(\frac{\pi}{4} - x\right)}{\sqrt{\sin x + \cos x}} dx$ is-

(A) $\frac{\pi\sqrt{2}}{4}$ (B) $\frac{\pi}{4\sqrt{2}}$
(C) 0 (D) None of these

Q.5 The value of $\int_{-2}^{3} |1 - x^2|\, dx$ is-

(A) $\frac{7}{3}$ (B) $\frac{14}{3}$ (C) $\frac{28}{3}$ (D) $\frac{1}{3}$

Q.6 The value of the integral $\int_{-2}^{3}\left[\cot^{-1}\left(\frac{x-1}{x+1}\right) + \cot^{-1}\left(\frac{x+1}{x-1}\right)\right]$ is-

(A) $\frac{5\pi}{2}$ (B) $\frac{3\pi}{2}$

(C) $\frac{\pi}{2}$ (D) None of these

Q.7 $\int_{-2}^{1} [x+1]\, dx$ is equal to-

(A) 0 (B) 1
(C) –1 (D) None of these

Q.8 $\int_0^2 \sin\frac{\pi[x]}{2} dx$ is equal to-

(A) 1 (B) –1
(C) 0 (D) None of these

Q.9 $\int_0^2 x^3\sqrt{2x - x^2}\, dx$ is equal to-

(A) $\frac{7\pi}{2}$ (B) $\frac{7\pi}{4}$ (C) $\frac{7\pi}{8}$ (D) $\frac{7\pi}{16}$

Q.10 $\int_{\pi/6}^{\pi/3} \frac{\sin x + \cos x}{\sqrt{\sin 2x}} dx$ is equal to-

(A) $2\sin^{-1}\frac{1}{2}(\sqrt{3} - 1)$

(B) $\sin^{-1}\frac{1}{2}(\sqrt{3} - 1)$

(C) $2\sin^{-1}\frac{1}{2}(\sqrt{3} + 1)$

(D) None of these

MATHEMATICS IIT JEE (OCT. 1st WEEK CLASS TEST 2) (DEFINITE INTEGRATION) ANSWER KEY

Name : ..... **Roll No. :** .....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○
										10	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	B	B	A	C	C	A	A	A	C	A

SOLUTIONS

Sol.1 (B)

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty}\left[\frac{1}{1+\sqrt{n}} + \frac{1}{2+\sqrt{2n}} + \ldots\ldots + \frac{1}{n+\sqrt{n^2}}\right]$$

$$= \lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{r+\sqrt{rn}} = \lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}\cdot\frac{1}{\frac{r}{n}+\sqrt{\frac{r}{n}}}$$

$$= \int_0^1 \frac{dx}{x+\sqrt{x}} = \int_0^1 \frac{2t}{t+t^2}dt = 2\int_0^1 \frac{dt}{1+t}$$

[Putting $x = t^2$ so that $dx = 2t\,dt$]

$$= 2\left[\log(1+t)\right]_0^1 = 2\log 2 = \log 4.$$

Sol.2 (B)

$$\int_{-\pi/2}^{\pi/2} \sin|x|\,dx = 2\int_0^{\pi/2} \sin|x|\,dx$$

$$= 2\int_0^{\pi/2} \sin x\,dx = -2\left[\cos x\right]_0^{\pi/2} = 2$$

Sol.3 (A)

Put $x = \cos\theta$ so that $dx = -\sin\theta\,d\theta$

$$\therefore\quad I = \int_{\pi/2}^{0} \cos\left[2\cot^{-1}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\right](-\sin\theta)d\theta$$

$$= \int_0^{\pi/2} \cos\left(2\cot^{-1}\tan\frac{\theta}{2}\right)\sin\theta\,d\theta$$

$$= \int_0^{\pi/2} \cos\left[2\cot^{-1}\cot\left(\frac{\pi}{2}-\frac{\theta}{2}\right)\right]\sin\theta\,d\theta$$

$$= \int_0^{\pi/2} \cos\left[2\left(\frac{\pi}{2}-\frac{\theta}{2}\right)\right]\sin\theta\,d\theta$$

$$= \int_0^{\pi/2} \cos(\pi-\theta)\sin\theta\,d\theta$$

$$= -\int_0^{\pi/2} \cos\theta\sin\theta\,d\theta$$

$$= \frac{-\sin^2\theta}{2}\Big|_0^{\pi/2} = \frac{-1}{2} \quad\therefore\quad I = \frac{-1}{2}$$

Sol.4 (C)

$$\text{Let } I = \int_0^{\pi/2} \frac{\left(\frac{\pi}{4}-x\right)}{\sqrt{\sin x+\cos x}}\,dx$$

$$= \int_0^{\pi/2} \frac{\frac{\pi}{4}-\left(\frac{\pi}{2}-x\right)}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)}}\,dx$$

$$= \int_0^{\pi/2} \frac{\frac{-\pi}{4}+x}{\sqrt{\sin x+\cos x}}\,dx$$

Adding, we get $2I = 0$, $\therefore\ I = 0$

Sol.5 (C)

$$\int_{-2}^{3} |1-x^2|\,dx$$

$$= \int_{-2}^{-1} (x^2-1)\,dx + \int_{-1}^{1} (1-x^2)\,dx + \int_1^3 (x^2-1)\,dx$$

$$= \left[\frac{x^3}{3}-x\right]_{-2}^{-1} + \left[x-\frac{x^3}{3}\right]_0^1 + \left[\frac{x^3}{3}-x\right]_1^3$$

$$= \frac{2}{3} + \frac{2}{3} + 2\left(\frac{2}{3}\right) + (9-3) - \left(\frac{1}{3}-1\right)$$

$$= \frac{10}{3} + 6 = \frac{28}{3}$$

Sol.6 (A)

$$\int_{-2}^{3}\left(\cot^{-1}\frac{x-1}{x+1}+\cot^{-1}\frac{x+1}{x-1}\right)\,dx$$

$$=\int_{-2}^{3}\left(\tan^{-1}\frac{x+1}{x-1}+\cot^{-1}\frac{x+1}{x-1}\right)\,dx$$

$$=\int_{-2}^{3}\frac{\pi}{2}dx=\frac{\pi}{2}[3-(2)]=\frac{5\pi}{2}.$$

Sol.7 (A)

$$\int_{-2}^{1}[x+1]\,dx=\int_{-2}^{-1}[x+1]\,dx$$

$$+\int_{-1}^{0}[x+1]\,dx+\int_{0}^{1}[x+1]\,dx$$

$[\because [x+1]=-1$ if $-2\le x<-1$; 0 if $-1\le x<0$; 1 if $0\le x<1]$

$$=\int_{-2}^{-1}(-1)\,dx+\int_{-1}^{0}0\,dx+\int_{0}^{1}1\,dx$$

$$=-(x)|_{-2}^{-1}+(x)|_{0}^{1}=0.$$

Sol.8 (A)

$$\int_0^2\sin\frac{\pi[x]}{2}\,dx=\int_0^1\sin\frac{\pi[x]}{2}dx+\int_1^2\sin\frac{\pi[x]}{2}dx$$

$$=\int_0^1\sin\frac{\pi.0}{2}\,dx+\int_1^2\sin\frac{\pi.1}{2}\,dx$$

$[\because [x]=0$ if $0\le x<1$, 1 if $1\le x<2]$

$$=0+x|_1^2=2-1=1.$$

Sol.9 (C)

$$\int_0^2 x^3\sqrt{2x-x^2}\,dx$$

$$\int_0^{\pi/2}8\sin^6\theta\,\sqrt{4\sin^2\theta-4\sin^4\theta}$$

$$(4\sin\theta\cos\theta)\,d\theta$$

[Putting $x=2\sin^2\theta$

$\Rightarrow dx=4\sin\theta\cos\theta\,d\theta$]

$$=64\int_0^{\pi/2}\sin^8\theta\,.\cos^2\theta\,d\theta$$

$$=64\,.\,\frac{7.5.3.1.1}{10.8.6.4.2}\,.\,\frac{\pi}{2}=\frac{7\pi}{8}.$$

Sol.10 (A)

$$\text{Let } I=\int_{\pi/6}^{\pi/3}\frac{\sin x+\cos x}{\sqrt{\sin 2x}}dx$$

$$=\int_{\pi/6}^{\pi/3}\frac{\sin x+\cos x}{\sqrt{1-(1-\sin 2x)}}\,dx$$

$$=\int_{\pi/6}^{\pi/3}\frac{\sin x+\cos x}{\sqrt{1-(\sin x-\cos x)^2}}\,dx$$

$$=\int_{\frac{1-\sqrt{3}}{2}}^{\frac{\sqrt{3}-1}{2}}\frac{dz}{\sqrt{1-z^2}}$$

[Putting $\sin x-\cos x=z$

$\Rightarrow (\cos x+\sin x)\,dx=dz$]

$$=\left[\sin^{-1}z\right]_{\frac{1-\sqrt{3}}{2}}^{\frac{\sqrt{3}-1}{2}}=2\sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right).$$