OCT. 1st WEEK CLASS TEST 3 DEFINITE INTEGRATION MATHEMATICS (FRESHER)

ABLES EDUCATIONS

Dear student following is a Moderate level [O ● O] test paper. Score of 12 Marks in 15 Minutes would be a satisfactory performance. Questions 1-8 (+3, –1). (Questions may have more than one option correct)

A special type of definite integral

$I(\alpha, \beta) = \int_0^1 2\sin(\alpha t)\sin(\beta t)\,dt$,

where $\alpha, \beta \in R - \{0\}$

Now this is integrated and then subjected to equate with zero

Two cases arises :

case I $\rightarrow \quad \alpha \neq \beta \Rightarrow \quad \alpha \neq 0$

and α_1, α_2 are two possible values of α for $I = 0$

β_1, β_2 are two possible values of β for $I = 0$

case II $\rightarrow \quad \alpha = \beta$

Q.1 For $\alpha = \beta$, what is the value of I.

(A) $\frac{\alpha^2}{1+\alpha^2}$ (B) $\frac{1}{1+\alpha^2}$

(C) $\frac{1}{|1-\alpha^2|}$ (D) $\left(\frac{\alpha}{1+\alpha^2}\right)$

Q.2 What is the range of I (α, α) for all $\alpha \in R$.

(A) (0, 1) (B) (0, 1]

(C) [0, 1) (D) [0, 1]

Q.3 If $(\alpha, \beta) \in [-\pi, \pi]$ and $\alpha > \beta$ and $I = 0$ then what is true ?

(A) $(\beta_1 + \pi/2)(\beta_2 + \pi/2) < 0$

(B) $\alpha + \beta = 0$

(C) $|\alpha| = |\beta|$

(D) $\left(\alpha_1 - \frac{\pi}{2}\right)\left(\alpha_2 - \frac{\pi}{2}\right) < 0$

Q.4 If $\alpha, \beta \in (0, 2\pi]$, and $I = 0$, then what is true ?

(A) $\frac{\pi}{2} < |\alpha - \beta| < \frac{3\pi}{2}$

(B) $\pi < |\alpha - \beta| < 2\pi$

(C) $\pi < \alpha + \beta < 3\pi$

(D) $\pi < \alpha + \beta < 2\pi$

Q.5 If $\alpha, \beta \in [-2\pi, 0)$ and $I = 0$, then what is true ?

(A) $\frac{\pi}{2} < |\alpha - \beta| < 3\pi/2$

(B) $\pi < |\alpha - \beta| < 2\pi$

(C) $-3\pi < \alpha + \beta < -\pi$

(D) $-2\pi < (\alpha + \beta) < -\pi$

Q.6 For $(\alpha, \beta) \in \left(\frac{-\pi}{2}, \pi/2\right)$ and $I = 0$ and $\alpha > \beta$, then what is the value of $m = \frac{\tan\alpha}{\alpha}$

(A) $m > 0$ (B) $m < 0$

(C) $m > 1$ (D) $m < 1$

Q.7 For $\alpha, \beta \in [-\pi, \pi]$ and $I = 0$ for $\alpha > \beta$, then all possible value of $m = \frac{\tan\alpha}{\alpha}$ is-

(A) $m \in (-\infty, 0]$ (B) $m \in R - (0, 1]$

(C) $m \in R$ (D) $m \in R^+$

Q.8 For $I = 0$ and $\alpha > \beta > 0$ $\alpha, \beta \in [0, 2\pi]$ then all possible value of $m = \frac{\tan\alpha}{\alpha}$ is-

(A) $m \in R$ (B) $m \in R^+$

(C) $m \in R - (0, 1]$ (D) $m \in R^-$

MATHEMATICS IIT JEE (OCT. 1st WEEK CLASS TEST 3) (DEFINITE INTEGRATION) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○
										10	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8
Ans.	A	C	All	A,C	A,C	C	B	B

SOLUTIONS

Given that $I = \int_0^1 2\sin(\alpha t)\ \sin(\beta t)\ dt$

(a) $I = \left[2\sin\beta r\left(\frac{\cos\alpha t}{\alpha}\right)\right]_0^1 - \int_0^1 2\beta\cos\ \beta t\left(\frac{-\cos\alpha t}{\alpha}\right)dt$ $\left[\begin{array}{l}\text{Integrating by parts, taking}\\ \sin\alpha t \text{ as second function}\end{array}\right]$

$\therefore\quad I = -\frac{2}{\alpha}\cos\ \alpha\ \sin\ \beta + \frac{2\beta}{\alpha}\int_0^1 \cos\beta t\cos\ \alpha t\ dt$

$= -\frac{2}{\alpha}\cos\ \alpha\ \sin\ \beta + \frac{2\beta}{\alpha}\left[\left[\cos\beta t\frac{\sin\alpha t}{\alpha}\right]_0^1 + \frac{\beta}{\alpha}\int_0^1 \sin\beta t\sin\alpha t\ dt\right]$

$I = -\frac{2}{\alpha}\cos\ \alpha\ \sin\ \beta + \frac{2\beta}{\alpha^2}\ \sin\ \alpha\ \cos\ \beta + \frac{\beta^2}{\alpha^2}\ I$

or $\quad I\left(1-\frac{\beta^2}{\alpha^2}\right) = \frac{2\beta}{\alpha^2}\ \sin\ \alpha\ \cos\ \beta - \frac{2}{\alpha}\ \cos\ \alpha\ \sin\ \beta = \cos\ \alpha\ \sin\ \beta\ \left[\frac{2\beta}{\alpha^2}\tan\alpha - \frac{2}{\alpha}\tan\beta\right]$

$\therefore\quad I = 0 \qquad (\because\ \tan\ \alpha = \alpha \text{ and } \tan\ \beta = \beta)$

(b) Given that $\alpha = \beta$

$I = \int_0^1 2\sin^2\ \alpha\ t\ dt = \int_0^1 (1-\cos 2\alpha t)\,dt$

$= \left[t - \frac{\sin 2\alpha t}{2\alpha}\right]_0^1 = 1 - \frac{\sin 2\alpha}{2\alpha}$

$= 1 - \frac{2\tan\alpha}{2\alpha(1+\tan^2\alpha)}$

$= 1 - \frac{2\alpha}{2\alpha(1+\alpha^2)} = \frac{\alpha^2}{1+\alpha^2}$

$I = 0 \quad \Rightarrow \quad \frac{\tan\alpha}{\alpha} = \frac{\tan\beta}{\beta}$

$\Rightarrow\quad y = \frac{\tan x}{x} = m$ has atleast two solutions.

CORRESPONDING GRAPHS $\Rightarrow$