Dear student following is an Easy level [● O O] test paper. Score of 24 Marks in 15 Minutes would be a satisfactory performance. Questions 1-10 (+3, –1). (All questions have only one option correct)

Q.1 The value of the integral $\int_0^{\pi} e^{\cos^2 x} \cdot \cos^3 (2n + 1)x \, dx$, n is integer, is-

(A) 0 (B) π (C) 2π (D) None

Q.2 $\int_0^{\pi/4} \frac{\sin^2 x.\cos^2 x}{(\sin^3 x + \cos^3 x)^2} dx$ is equal to-

(A) $\frac{1}{6}$ (B) $\frac{1}{12}$ (C) $\frac{1}{4}$ (D) None

Q.3 The value of the integral $\int_0^r xy \, dx$, where $y = \sqrt{r^2 - x^2}$, is-

(A) $\frac{r^3}{6}$ (B) $\frac{r^3}{3}$ (C) $\frac{r^3}{2}$ (D) None

Q.4 $\int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$ is equal to-

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{12}$ (D) None

Q.5 $\int_0^{\pi/2}$ log sin x dx is equal to-

(A) $\frac{\pi}{2}$ log 2 (B) $-\frac{\pi}{2}$ log 2

(C) π log 2 (D) None of these

Q.6 The value of the integral $\int_0^{\pi}$ x log sin x dx is-

(A) $-\frac{\pi^2}{2}$ log 2 (B) $\frac{\pi^2}{2}$ log 2

(C) π^2 log 2 (D) None of these

Q.7 The value of the integral $\int_0^{\pi}$ log (1 + cos x) dx is-

(A) $\frac{\pi}{2}$ log 2 (B) π log 2

(C) – π log 2 (D) None of these

Q.8 The value of the integral $\int_0^{\pi} \frac{\sin 2kx}{\sin x} dx$, where $k \in I$, is-

(A) $\frac{\pi}{2}$ (B) π (C) 0 (D) None

Q.9 $\int_{-1}^{1} \log \frac{2-x}{2+x} dx$ is equal to-

(A) –1 (B) 1 (C) 2 (D) 0

Q.10 $\int_{-1}^{1} [x] \, dx$, where [.] denotes the greatest integer function, is equal to-

(A) 0 (B) 1

(C) –1 (D) None of these

MATHEMATICS IIT JEE (SEPT. 5th WEEK CLASS TEST 2) (DEFINITE INTEGRATION) ANSWER KEY

Name : ..... **Roll No. :** .....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○
										10	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	A	A	B	C	B	A	C	C	D	C

SOLUTIONS

Sol.1 (A)

Let $I = \int_0^{\pi} e^{\cos^2 x} \cdot \cos^3 (2n + 1)\, x\, dx$

$= \int_0^{\pi} e^{\cos^2(\pi - x)} \cdot \cos^3 (2n + 1)\, (\pi - x)\, dx$

$= \int_0^{\pi} e^{\cos^2 x} \cdot \cos^3 [(2n+1)\pi - (2n+1)x]\, dx$

$= -\int_0^{\pi} e^{\cos^2 x} \cos^3 (2n + 1)x\, dx = -1$

$\therefore \quad 2I = 0 \Rightarrow I = 0.$

Sol.2 (A)

Let $I = \int_0^{\pi/4} \frac{\sin^2 x . \cos^2 x}{(\sin^3 x + \cos^3 x)^2}\, dx$

$= \int_0^{\pi/4} \frac{\tan^2 x . \sec^2 x}{(1 + \tan^3 x)^2}\, dx$

$= \int_1^2 \frac{dz}{3z^2}$ [Put $1 + \tan^3 x = z$

$\Rightarrow \tan^2 x \sec^2 x\, dx = 1/3\, dz$]

$= \left[\frac{-1}{3z}\right]_1^2 = -\frac{1}{3}\left(\frac{1}{2} - 1\right) = \frac{1}{6}.$

Sol.3 (B)

Let $I = \int_0^r xy\, dx = \int_0^r x\sqrt{r^2 - x^2}\, dx$

$= \int_{r^2}^0 \frac{-1}{2}\sqrt{z}\, dz$

[Put $r^2 - x^2 = z \Rightarrow x\, dx = \frac{-1}{2} dz$]

$= \frac{1}{2}\int_0^{r^2} \sqrt{z}\, dz = \left[\frac{z^{3/2}}{3}\right]_0^{r^2} = \frac{1}{3}(r^3)$

Sol.4 (C)

Let $I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\, dx$

$= \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin\left(\frac{\pi}{6} + \frac{\pi}{3} - x\right)}}{\sqrt{\sin\left(\frac{\pi}{6} + \frac{\pi}{3} - x\right)} + \sqrt{\cos\left(\frac{\pi}{6} + \frac{\pi}{3} - x\right)}}\, dx$

$= \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}\, dx$

$\therefore \quad 2I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}\, dx$

$= [x]_{\pi/6}^{\pi/3} = \frac{\pi}{6}$

$\therefore \quad I = \frac{\pi}{12}.$

Sol.5 (B)

Sol.6 (A)

Let $I = \int_0^{\pi} x \log \sin x\, dx$

Applying formula $\int_a^b f(x)\, dx = \int_a^b f(a + b - x)\, dx$

We get $I = \frac{-\pi^2}{2} \log 2.$

Sol.7 (C)

Let $I = \int_0^{\pi} \log (1 + \cos x)\, dx$

Applying formula $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

and using the result

$$\int_0^{\pi/2} \log \sin x\, dx = \frac{-\pi}{2} \log 2$$

We get $I = -\ \pi \log 2$.

Sol.8 (C)

Let $I = \int_0^{\pi} \frac{\sin 2kx}{\sin x}\, dx = \int_0^{\pi} \frac{\sin 2k(\pi - x)}{\sin(\pi - x)}\, dx$

$$= \int_0^{\pi} \frac{\sin(2k\pi - 2kx)}{\sin x}\, dx$$

$$= -\int_0^{\pi} \frac{\sin 2kx}{\sin x}\, dx = -I$$

$\therefore\ 2I = 0 \Rightarrow I = 0.$

Sol.9 (D)

Let $f(x) = \log \left(\frac{2-x}{2+x}\right)$,

then $f(-x) = \log \left(\frac{2+x}{2-x}\right)$

$$= -\log \left(\frac{2-x}{2+x}\right) = -f(x)$$

So, the function f(x) is odd.

$\therefore\ \int_{-1}^{1} \log \frac{2-x}{2+x}\, dx = 0.$

Sol.10 (C)

$$\int_{-1}^{1} [x]\, dx = \int_{-1}^{0} (-1)\, dx + \int_0^1 0\, dx$$

$[\because [x] = -1 \text{ if } -1 \le x < 0$

$0 \text{ if } 0 \le x < 1]$

$$= -(x)|_{-1}^{0} = -1$$