Dear student following is an Easy level [● O O] test paper. Score of 24 Marks in 15 Minutes would be a satisfactory performance. Questions 1-10 (+3, –1). (All questions have only one option correct)

Q.1 $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}} =$

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{12}$ (D) $\frac{\pi}{4}$

Q.2 If $I_1 = \int_{-4}^{-5} e^{(x+5)^2}\, dx$ and

$I_2 = \int_{1/3}^{2/3} e^{9(x-2/3)^2}\, dx$ then $I_1 + 3I_2 =$

(A) 0 (B) – 1 (C) 3 (D) None

Q.3 The value of $\int_0^1 x(1-x)^{98}\, dx$ is-

(A) $\frac{1}{9900}$ (B) $\frac{1}{4995}$

(C) $\frac{1}{9997}$ (D) None of these

Q.4 If $\int_{\pi/2}^{x} \sqrt{3-2\sin^2 t}\, dt + \int_0^y \cos t\, dt = 0$,

then $\left(\frac{dy}{dx}\right)$ at $x = \pi$ and $y = \pi$ is-

(A) $\sqrt{3}$ (B) $-\sqrt{2}$

(C) $-\sqrt{3}$ (D) None of these

Q.5 If f(x) is an integrable function in $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$

and $I_1 = \int_{\pi/6}^{\pi/3} \sec^2\theta\, f(2\sin 2\theta)\, d\theta$ and

$I_2 = \int_{\pi/6}^{\pi/3} \text{cosec}^2\theta\, f(2\sin 2\theta)\, d\theta$, then-

(A) $I_1 = 2I_2$ (B) $I_1 = 3I_2$
(C) $2I_1 = I_2$ (D) None of these

Q.6 If $I = \int_{-1}^{1} \left(\left[x^2\right] + \log\left(\frac{2+x}{2-x}\right)\right) dx$

where [x] denotes the greatest integer $\leq$ x, then I equals-

(A) – 2 (B) – 1 (C) 0 (D) 1

Q.7 If $I = \int_{-\pi}^{\pi} \frac{\sin^2 x}{1+a^x}\, dx$, a > 0, then I equals-

(A) π (B) $\frac{\pi}{2}$ (C) $a\pi$ (D) $\frac{a\pi}{2}$

Q.8 If $I = \int_{-3}^{2} (|x+1| + |x+2| + |x-1|)dx$, then I equals-

(A) $\frac{31}{2}$ (B) $\frac{35}{2}$ (C) $\frac{37}{2}$ (D) $\frac{39}{2}$

Q.9 If $I = \int_{-2}^{2} |1-x^4|\, dx$, then I equals-

(A) 6 (B) 8 (C) 12 (D) 21

Q.10 If $I = \int_0^{\pi} \sin^3 x\, (1 + 2\cos x)\, (1+\cos x)^2\, dx$, then I equals-

(A) $\frac{4}{3}$ (B) $\frac{2}{3}$ (C) $\frac{8}{3}$ (D) 2

MATHEMATICS IIT JEE (SEPT. 5th WEEK CLASS TEST 3) (DEFINITE INTEGRATION) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	⬭	⬭	⬭	⬭	4	⬭	⬭	⬭	⬭	7	⬭	⬭	⬭	⬭
2	⬭	⬭	⬭	⬭	5	⬭	⬭	⬭	⬭	8	⬭	⬭	⬭	⬭
3	⬭	⬭	⬭	⬭	6	⬭	⬭	⬭	⬭	9	⬭	⬭	⬭	⬭
										10	⬭	⬭	⬭	⬭

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	C	A	A	A	D	C	B	A	C	C

SOLUTIONS

Sol.1 (C)

Let $I = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$

$= \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\, dx$

$\Rightarrow I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}\, dx$

$\therefore\ 2I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}+\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}\, dx$

$= \int_{\pi/6}^{\pi/3} dx = (x)_{\pi/6}^{\pi/3}$

$= \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$

$\Rightarrow I = \frac{\pi}{12}$

Sol.2 (A)

$I_1 = \int_{-4}^{-5} e^{(x+5)^2}\, dx$

$= (-5+4) \int_0^1 e^{[(-5+4)x-4+5]^2}\, dx$

$= -\int_0^1 e^{(x-1)^2}\, dx$

and $I_2 = \int_{1/3}^{2/3} e^{9\left(x-\frac{2}{3}\right)^2}\, dx$

$= \left(\frac{2}{3}-\frac{1}{3}\right) \int_0^1 e^{9\left[\left(\frac{2}{3}-\frac{1}{3}\right)x+\frac{1}{3}-\frac{2}{3}\right]^2}\, dx$

$= \frac{1}{3}\int_0^1 e^{(x-1)^2}\, dx$

$\Rightarrow I_1 + 3I_2 = 0$

Sol.3 (A)

$I = \int_0^1 x(1-x)^{98}\, dx = \int_0^1 (1-x)\, x^{98}\, dx$

$\because\ \int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx$

$= \frac{1}{99} - \frac{1}{100} = \frac{1}{9900}$

Sol.4 (A)

$\int_{\pi/2}^{x} \sqrt{3-2\sin^2 t}\, dt + \int_0^y \cos t\, dt = 0$, then

$\left(\frac{dy}{dx}\right)$ at $x = \pi$ and $y = \pi$.

$\left(\sqrt{3-2\sin^2 t}\right) .\ 1 + \cos y\ \frac{dy}{dx} = 0$

$\Rightarrow \left(\frac{dy}{dx}\right)_{\pi,\ \pi} = \frac{-\sqrt{3-2\sin^2 x}}{\cos y} = \sqrt{3}$.

Sol.5 (D)

$I_1 = \int_{\pi/6}^{\pi/3} \sec^2\theta\ f(2\sin 2\theta)\, d\theta$

Apply property $\int_a^b f(a+b-x)\, dx$

$I_1 = \int_{\pi/6}^{\pi/3} \sec^2\left(\frac{\pi}{2}-\theta\right) f\left(2\sin 2\left(\frac{\pi}{2}-\theta\right)\right) d\theta$

$I_1 = \int_{\pi/6}^{\pi/3} \mathrm{cosec}^2\theta\ f(2\sin 2\theta)\, d\theta = I_2$

Sol.6 (C)

As $\log\left(\frac{2+x}{2-x}\right)$ is an odd function, we can write

$$I = \int_{-1}^{1} [x^2]\, dx + 0$$

But for $-1 < x < 1$, $0 \le x^2 < 1$ and thus, $[x^2] = 0$

$\therefore\ I = 0$

Sol.7 (B)

$$I = \int_{-\pi}^{\pi} \frac{\sin^2 x}{1+a^x}\, dx \quad \text{......... (1)}$$

$$I = \int_{-\pi}^{\pi} \frac{(\sin(-x))^2}{1+a^{-x}}\, dx$$

$$= \int_{-\pi}^{\pi} \frac{a^x \sin^2 x}{1+a^x}\, dx \quad \text{......... (2)}$$

Adding (1) and (2), we get

$$2I = \int_{-\pi}^{\pi} \sin^2 x\, dx$$

$$= 2\int_{0}^{\pi} \sin^2 x\, dx$$

$$= \int_{0}^{\pi} (1 - \cos 2x)\, dx$$

$$= \left(x - \frac{\sin 2x}{2}\right)_0^{\pi} = \pi$$

$$\Rightarrow I = \frac{\pi}{2}.$$

Sol.8 (A)

We can write

$$I = I_1 + I_2 + I_3$$

Where $I_1 = \int_{-3}^{2} |x + 1|\, dx$ etc.

Put $x + 1 = t$, so that

$$I_1 = \int_{-2}^{3} |t|\, dt = \int_{-2}^{0} (-t)\, dt + \int_{0}^{3} t\, dt$$

$$= -\frac{1}{2}\left.t^2\right]_{-2}^{0} + \left.\frac{1}{2}t^2\right]_0^3 = \frac{13}{2}$$

Similarly, $I_2 = I_3 = \frac{9}{2}$

Thus, $I = \frac{31}{2}$

Sol.9 (C)

As $|1 - x^4|$ is an even function

$$I = 2\int_{0}^{2} |1 - x^4|\, dx$$

$$\therefore\ I = 2\int_{0}^{1} (1 - x^4)\, dx + 2\int_{1}^{2} (x^4 - 1)dx$$

$$= 2\left.\left(x - \frac{x^5}{5}\right)\right]_0^1 + 2\left.\left(\frac{x^5}{5} - x\right)\right]_1^2$$

$$= 2\left(1 - \frac{1}{5}\right) + 2\left(\frac{32}{5} - 2 - \frac{1}{5} + 1\right)$$

$$= 12$$

Sol.10 (C)

Put $\cos x = t$, so that

$$I = \int_{-1}^{1} (1 - t^2)(1 + 2t)(1 + t)^2\, dt$$

$$= \int_{-1}^{1} (1 - t^2)(1 + 4t + 5t^2 + 2t^3)dt$$

$$= 2\int_{0}^{1} (1 - t^2)(1 + 5t^2)\, dt$$

[$\because$ remaining part of integrand is an odd function]

$$= 2\int_{0}^{1} (1 + 4t^2 - 5t^4)\, dt$$

$$= 2\left.\left(t + \frac{4}{3}t^3 - t^5\right)\right]_0^1$$

$$= 2\left(1 + \frac{4}{3} - 1\right) = \frac{8}{3}.$$